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SYMPOSIUM ON THE EARTH: ITS FIGURE, DIMENSIONS AND THE CONSTITUTION OF ITS INTERIOR.

I.

THE INTERIOR OF THE EARTH FROM THE VIEWPOINT OF GEOLOGY.[1]

BY T. C. CHAMBERLIN.

(*Read April 24, 1915.*)

For some time past there has been a marked drift of geologic opinion from the older tenet of a molten earth toward the conviction that the earth is essentially solid. This has been quite as much due to the contributions of kindred sciences as to the growth of geologic evidence, but this has made its important and concurrent contributions.

The great granitic embossments that constitute the most distinctive feature of the oldest known terranes were formerly regarded as solidified portions of a primitive molten earth and seemed to serve as witnesses of the verity of the former liquid state. A few years ago, however, it was determined—almost simultaneously in several countries where critical studies on these formations were

[1] The discussion of this topic at the session of the Society was without manuscript or notes and this paper, prepared some weeks later, is less a reproduction of the original discussion than a substitute for it.

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in progress—that these granitic masses are not only intrusive but that they were thrust into formations that had previously been formed at the surface of the earth. These surface formations have thus come to stand as the most ancient terranes now known. These earliest accessible depositions imply the preëxistence of a substantial foundation formed at a still earlier date. Neither of these gives any clear intimation that lower formations are different from themselves. So far then as the record runs back, it testifies to substantial solidity in the outer part of the globe at least. The record implies, indeed, that molten matter was then present within the earth, but it gives no certain measure of the ratio of the molten to the solid part. There is no determinate evidence that a molten condition was a preponderant state, even in the interior, at any stage covered by the lithographic record. The interior conditions of the earliest stages that antedate the lithographic record are to be reached only by indirect and remote rather than direct and immediate inference. Under the influence of inherited presumptions, it may seem to many still probable that the interior of the mature earth was once dominated by a molten condition at some remote stage, but the phenomena of powerful inthrusting, so often shown in the intrusions of the igneous element into the early terranes, seems to imply that at the Archean stages the molten element was in the strong grasp of such stresses as are natural to a rigid globe and was therefore then but a minor and passive factor, not a controlling one.

When it is considered that, if the earth were once wholly molten, the material for all the stratified rocks of later ages must have been derived from the primitive crust after it was formed and forced into positions of erosion—or from matter extruded through it—the absence, according to present knowledge, of any great area of rocks bearing the distinctive characteristics of the congealed surface greatly weakens the assumption that the postulated molten state ever obtained in the mature earth.

A study of the stress-conditions of the interior of the earth seems to call for a similar reversal of the inferences once drawn from the igneous rocks. From the earliest well-recorded ages, the exerior of the earth has given evidence of broad topographic reliefs

in the form of great embossments and basins. These surface configurations must have conditioned the localization of extrusions and the deployment of the effusive material. If the lavas arose from a general and abundant source of supply which was responsive to general and powerful stresses, vestiges of this vital relation should be found in the volume and deployment of the lava floods. If, on the other hand, the molten material was but a fraction of the enviroining mass, variously distributed through it, the result should be a multitude of driblets squeezed out here and there in such special situations as the controlling stresses required, or else forced into weak portions of the earth-body where the stresses were less imperative. Now there is abundant geological evidence that the earth-body has been subjected at repeated intervals to strong compressive stresses by which its outer portion has been folded into mountainous ranges, or pushed up into great plateaus, while masses of continental dimensions have been raised, relatively, to notable heights, and the bottoms of basins and deeps have sunk reciprocally to even greater relative depths. The internal stresses which these deformations imply should have made themselves felt proportionately on any great mass of liquid in the interior—if it were in existence—and extrusions proportionate to the great deformations of the rigid material should have accompanied such diastrophism. But, while liquid extrusions took place somewhat freely at the times of great diastrophism, it was not, at least in my judgment, at all commensurate with the deformative stresses implied by the diastrophic results in the solid material.

Nor was the concentration of the extrusions indicative of origin from a molten interior or from great residual reservoirs of liquid rock. If such ample sources of liquid had existed they might naturally have been expected to have given forth, under the great stresses then seeking easement, correspondingly great floods of lava. Yet no single lava flood seems to have attained more than an extremely small fraction of the mass of the earth or of the known solid matter of its region. Even when the sum total of the most massive series of successive floods in a given region are taken together—though the successive issues stretched over a considerable period—they rarely rise above a most insignificant fraction of earth-mass or even of the

regional segment of it with which they are associated. Instead of really massive flows, implying ample sources of supply and great forces of extrusion, the record shows rather a multitude of little ejections or injections of more or less sporadic distribution. The logical implication of these is the preëxistence of a multitude of small liquid spots, or liquifiable spots, scattered widely through the stressed earth-masses and yielding to stress as local conditions required.

This inference is supported by the great variations in altitude at which lavas are given forth. The most impressive illustrations of this are found in current volcanic action whose relations in altitude are precisely known. So far as ancient conditions can be restored, they appear to fall into the same general class as existing conditions. Current outpourings of lava range from the sea bottom to altitudes of many thousands of feet above sea level, a vertical range of several miles. Extrusions occur at these significantly diverse altitudes simultaneously or alternately or in almost any time-relations, and sometimes in the most marked independence of one another in spite of the natural sympathy of such events in a common stressed body. A multitude of facts of detail, some of which are singularly cogent, imply that the lava sources of present volcanoes are disconnected from one another in the interior, and hence independent in action, as a rule, though sometimes they show sympathy without showing liquid connection. The sources of lava seem to be meager in general, and the eruptive agencies seem to be controlled by narrowly local conditions. There is an absence of evidence that the lavas in the craters and necks of volcanoes are parts of great liquid masses below, responsive to the common stresses of a large region.

Thus geological evidence, when critically scrutinized, seems to be distinctly adverse to the existence of even large reservoirs of molten matter within the earth; it points rather to the presence of scattered spots, very small relatively, on the verge of liquefaction, which pass by stages into the liquid form and are then forced out by the differential stresses that abound in the earth body, each such local liquifying center commonly giving forth driblets of lava and gas, at intervals, none of which often rise to more than an extremely minute fraction of the earth mass or even of the subterranean mass contiguous to the volcano.

A revised view of the nature and location of earth-stresses seems also to be required by what is now known of earth-conditions. Under the former dominance of the tenet of a molten globe, it was natural to assign to the stress-differences of the earth a distinctly superficial localization and limitation; they were thought to be affections of "the crust" almost solely. Hydrostatic pressures were of course recognized as affecting the deep interior, but these were obviously balanced stresses, they were ineffective in deformation. The stresses supposed to give rise to the great reliefs of the earth's surface were thought to be very superficial. But the stresses imposed by known deformative agencies are not all superficial, nor are their intensities always greatest at the surface. According to Sir George Darwin, the stress-differences generated in the earth by the tidal forces of the moon are eight times as great at the center of the earth as at the surface. So also, according to the same authority, the stresses engendered by changes in the rotation of the earth are eight times as great at the center as at the surface and are graded between center and surface. The tidal stress-differences are relatively feeble but are perpetually renewed in pulsatory fashion. Those that arise from rotation belong to the highest order of competency. The stress-difference that would arise at the center of the earth from a stoppage of the earth's rotation, would, according to Darwin, reach 32 tons per square inch. Changes of the rate of rotation are almost inevitable when great diastrophic readjustments take place. Such periods are to be regarded as critical times at which great floods of lava should be poured forth from the interior if liquid material were there in great volume ready to respond to the changes of capacity which the deformation of the earth's sectors and the change in the spheroidal form would inevitably impose.

Not to detain you with other considerations, the foregoing seem best to comport with an essentially solid state of the earth's interior, if they do not point rather definitely to such a state. Even if they stood alone, they would seem to make a prevailing solid state the most tenable working hypothesis.

But they are far from standing alone; the geological evidences are strongly supported by considerations that spring from several kindred lines of inquiry. The testimony of astronomic evidence

has been given by Dr. Schlesinger. The import of seismic studies, the subject of Dr. Reid's contribution, lends very special support to the view that the interior of the earth is elastico-rigid at least to the extent that distortional waves have been shown to pass through its interior. It seems certain already that this condition prevails throughout much more than half the volume of the earth; concerning the rest, the deep interior, the seismic evidence is perhaps still to be regarded as indeterminate. But on the seismic evidence it does not fall to me to dwell.

The tidal studies of Hecker, Orloff and others lend support to the tenet of a rigid earth but they fall somewhat short of conclusiveness. The brilliant experimental determinations of Michelson and Gale, correlated with the computations of Moulton, have carried the evidence to the point of preliminary demonstration. They need only to be adequately repeated and verified to become final, so far at least as elastic rigidity can be indicated by the response of the earth-body to solar and lunar attractions. The special feature of most critical value in the demonstrations of Michelson and his colleagues is the high degree of elasticity shown by the almost instantaneous response of the earth to the distorting pull of the tide-producing bodies. This cuts at the very base of concepts founded on the supposed properties of a viscous earth. These tidal determinations of elasticity are in close accord with the seismic evidences of elasticity. The two are happily complementary to one another. The one deals with the earth as a whole under rhythmical series of increasing and diminishing stress-differences springing from external attraction; the other deals in an intensive partitive way with earth substance by sharp short stresses that call into action its most intimate structural qualities. While it is wise, no doubt, to refrain from resting too much on these early results of relatively new and radical lines of inquiry, until their results shall be more mature, their prospective import is radical and decisive in favor of a solid earth not only, but of an elastico-rigid earth. Assuming that the present import of these inquiries will be amply justified by more mature research, it is pertinent to bring into consideration the corollary they so distinctly imply, viz.: that the molten and viscous material in the earth, or at least in its outer half, if not throughout

its deep interior, is negligible in general studies, and enters into general terrestrial mechanics only as a subsidiary feature. It seems necessary to limit liquid and viscous lacunæ—if there are lacunæ in any proper sense at all—to such moderate dimensions that they do not seriously kill out distortional waves passing through the outer half of the globe in various directions; for seismic instruments show that these waves retain their integrity with surprising tenacity through long traverses. It seems equally necessary to limit the liquid and viscous factor rather severely if the interior structure is to be consistent with so prompt a response of the earth to twelve-hour stress-pulses as to imply almost complete elastic fidelity.

In the light of these determinations, strengthened not a little by their concurrence with the later geological determinations, the working hypotheses of the earth-student can scarcely fail to give precedence to dynamic tenets founded on a rigid earth.

The limitation of liquid and viscous matter, thus imposed, quite radically conditions all tenable views of magmas and of vulcanism, and thus bears upon the igneous nature of the interior. No small part of petrologic effort in past decades has been spent on the differentiation of magmas. To a notable degree these efforts have proceeded on the assumption, conscious or unconscious, that differentiation took its departure from an original homogeneous magma such as might arise from residual portions of a molten earth. Indefinite lapses of time, and such conditions of quiet as are naturally assignable to residual reservoirs of lava, have been freely assumed as working conditions without much question as to their reality. Under the hypothesis of a molten earth passing slowly into a partially solid earth, and retaining residual lacunæ of molten matter as an incident of the change, these assumptions are quite natural. On the other hand, under the hypothesis of a pervasively rigid earth, affected by stress-conditions that are constantly varying in intensity and in distribution—and subject to more radical changes at times of periodic readjustment—the existence of such residual magmas becomes at least questionable, perhaps improbable. Still more questionable is the assumption that the multitude of little liquid spots supposed to arise within the elastico-rigid mass, always have conformed to one type or set of types. The inherent proba-

bilities of the case seem to point strongly to wide variation in nature due to selective solution or differential fusion. The liquefying action that brings magmas into being, under this view, is presumably controlled by the same chemical and physical principles as the solidifying phases of the same cycle. The logical presumption is that at all stages of a magma's career from its inception through its growth, climax and decline to its final solidification, selective action will be in progress more or less and that no stage will be entitled to be regarded as original or parental in a special sense, such a sense for example as might be appropriate if the lava were the residue of an inherited original state and were merely differentiated by fractional crystallization as it passed toward solidification.

While these contrasted views of the history of magmas are naturally connected with views of the genesis of the earth, they are not limited to this relation. They are inherent in the very relations of solid and liquid matter and have a more or less important place irrespective of the earth's genesis.

An element of no small importance to a revised concept of the interior of the earth has arisen from geodetic studies on the distribution of densities within the earth. As the geodetic point of view is to be presented by its foremost exponent, Dr. Hayford, it is permissible for me merely to refer to certain geologic bearings.

On the assumption that the earth was once in a molten state, the inference is unavoidable that a perfect state of isostatic equilibrium was originally assumed by the surface, and that its configuration was at first strictly spheroidal. The material must have been arranged in concentric layers according to specific gravity and each layer should have had the same density at every point. All such reliefs of the earth's surface, and all such differences of specific gravity in the same horizon as have since arisen, must have been superinduced upon this originally perfect isostatic surface. With good reason therefore these inequalities have heretofore been supposed to be relatively shallow. On the hypothesis that the earth grew up by heterogeneous accretions, it is an equally natural inference that differences of specific gravity extend to great depths. In an endeavor to find out the bearings of geodetic data on the distribution of densities, Dr. Hayford tested four assumptions, all of

which he found measurably compatible with his geodetic data. From these he derived the respective depths of 37, 76, 109 and 179 miles as the horizons to which differences of density extended and below which they vanished or became negligible. Now all these depths are greater than had been assigned for probable differentiation in the traditional molten earth. On the other hand, the highest figure, 179 miles, was derived from a curve drawn specifically to represent the probable distribution of densities in an earth of planetesimal growth. The distribution represented by this highest figure fits the geodetic data quite as well as either of the other assumptions of distribution, though drawn on a strictly naturalistic basis If it could be said that geodetic data demonstrate that the actual differentiation of specific gravities has its sensible limits somewhere between 37 and 179 miles below the surface, such considerable depth would distinctly favor an accretionary origin as against a molten origin. But a conclusive determination is yet to be reached by geodetic inquiries.

While it is possible, within the broad terms of the planetesimal hypothesis, to suppose that the rate of accretion was so fast as to give rise to a molten planet, such a result seems to me extremely improbable under the actual conditions of the case. The growing planet should have become capable of holding a considerable atmosphere by the time it attained one tenth of its present mass, *i. e.*, about the mass of Mars. After this the protective cushion of the atmosphere should have greatly checked the plunge of the planetesimals and largely dissipated them into dust in the upper atmosphere where the inevitable heat of impact would be promptly radiated away. The dust presumably floated long and came gently to earth, so that, while the total heat generated by impact was large, the temperature of the earth body was probable never very high during the later stages of growth, and perhaps not at any stage of growth. Following out as well as may be the probable rates and conditions of growth, the most tenable concept of the state of the earth's interior under the planetesimal hypothesis is as follows:

The condition of the nuclear portion supposed to be formed from one of the knots of the parent spiral nebula and constituting a minor fraction of the mass of the earth, say thirty or forty per cent., is

left indeterminate by present lack of knowledge of the physical state of the knots of spiral nebulæ. If these are gaseous—which is rendered doubtful by their lack of strict sphericity—the nucleus was doubtless originally molten. If the constituents of the knot were held in orbital relations, their aggregation might have been slow enough to permit a solid state of even this portion. The matter added to the nucleus as planetesimal dust, or as planetesimals reduced in mass and speed by the atmosphere, probably retained its solid condition, with negligible exceptions, throughout the process of accretion except as selected portions passed into the liquid state and became subject to extrusive action. An intimate heterogeneity naturally prevailed throughout the whole mass so aggregated. A selective process, however, probably brought in the heavier matter faster and earlier than the lighter matter, for the magnetism of the earth should have aided gravity in gathering in the magnetic metals while the inelastic planetesimals, predominantly the heavy basic ones, when in collision destroyed the opposing components of their motions and hence yielded to the earth's gravity sooner than the more elastic ones. Relatively high specific gravity in the material of the deep interior is thought to have arisen at the outset and to have been increased by the selective vulcanism that came into action as growth proceeded. Special emphasis is laid on the *selective* nature of vulcanism under this hypothesis. The intimate mixture of planetesimals and planesesimal dust gave rise to a multitude of minute contacts between particles of different chemical and physical properties and hence there arose wide differences in the solution points. As the temperature in the growing planet rose, the more soluble portions passed into the liquid state by stages long before the remaining larger portion reached the temperature of solution. In a stressed globe certain of whose stresses are more intense toward the center than toward the surface, the solutions worked in the direction of least resistance, for them generally outwards, carrying heat of liquefaction and leaving the less soluble larger portion behind with temperatures inadequate for further liquefaction until there was a renewed accession of heat. The mechanism thus automatically tended to remove the most soluble constituents by progressive stages, while it tended to preserve the solid condition of the

main mass. The hypothesis thus supplies a working mechanism whose results fall into full accord with the states of the interior implied by tidal investigations and by seismic data, while the postulated distribution of specific gravities accords fairly well with geodetic determinations, as they now stand.

The adaptation of such an earth to isostatic adjustment can scarcely be more than hinted at here. The growth of the earth should have given it a concentric structure, while its highly distributive vulcanism, together with some of its deformative processes, should have given a vertical or radial structure, the two conjoining to give a natural tendency to prismatic or pyramidal divisions converging toward the center. The most powerful of all the deformative agencies, rotation, required for the adaptation of the earth to its changes of rate, such divisions of the earth-body as would respond most readily to depression in the polar and bulging in the equatorial tracts reciprocally. As urged elsewhere, this accommodation seems best met by three pyramidal sectors in each hemisphere with apices at the center and bases at the surface, the sectors in opposite hemispheres arranged alternately with one another. Very simple motions of these sectors on their apices at the earth's center would satisfy the larger demands of rotational distortion, while the sub-sectors into which these major sectors would naturally divide, as stresses required, would easily accommodate the nicer phases of adjustment. This primitive segmentation to meet rotational demands—which were most urgent during the stages of infall—furnished a mechanism suitable for the easement also of a portion of the deformational stresses that arose from other sources, among them gravitative stresses arising from loading and unloading by erosion and sedimentation. A gravitational adjustment by the wedging up and down and laterally of such sectors is thus offered tentatively as a working competitor to theories of adjustment by fluidal or quasi-fluidal undertow. The necessary brevity of this statement leaves this new hypothesis little more than a crude suggestion that gravitative adjustment (= isostasy) may perhaps take place as fully as the case requires in a highly rigid elastic earth without resort to flowage or even quasi-flowage.

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II.

CONSTITUTION OF THE INTERIOR OF THE EARTH AS INDICATED BY SEISMOLOGICAL INVESTIGATIONS.

By HARRY FIELDING REID.

(*Read April 24, 1915.*)

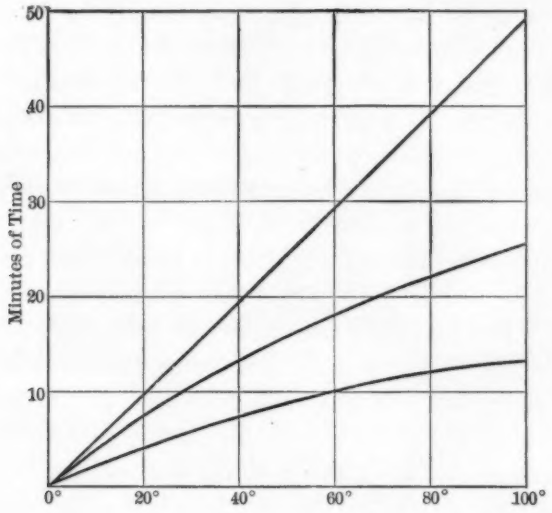
In 1883 Milne predicted that earthquake disturbances would be registered by seismographs at great distances from their origin, a prediction first verified when the earthquake of April 18, 1889, whose origin lay off the coast of Japan, affected the horizontal pendulum which von Rebeur-Paschwitz had set up at Potsdam to study the attraction of the moon. Milne was so convinced of the correctness of his idea and of the importance of the results to be obtained that in 1893 he established an observatory on the Isle of Wight to record earthquakes from distant regions; and he also succeeded in having instruments of similar model set up at observatories very widely scattered in various parts of the world.

Wertheim in 1851 showed that a disturbance in the interior of an elastic solid would break up into two groups of waves, longitudinal and transversal, which would be propagated at different rates, and as their velocities are so great that they cannot be separated from each other in the laboratory he suggested with rare insight that their separation might first be noticed in connection with the propagation of earthquake disturbances.[1] A few years later Lord Rayleigh showed that a third kind of wave could be propagated along the surface of the earth.[2] Seismologists naturally looked for indications of these three groups of waves in their

[1] "Sur la propagation du movement dans les corps solides et liquides," *Ann. de Chimie et Phys.*, 1851, Vol. XXI., p. 19.

[2] "On Waves Propagated along the Plane Surface of an Elastic Solid," *Proc. London Math. Soc.*, 1855, Vols. XLVII., L.

seismograms, but it was not until 1900 that Oldham succeeded in showing definitely that the seismograms of a number of Milne instruments gave clear evidence of the existence of three groups of waves. Oldham also published a diagram, which was an extension of Seebach's so-called "hodograph," showing the relation between the time of transmission of each group and the distance from the earthquake origin, measured along the surface of the earth. Milne soon improved these curves by adding observations of a large number of recorded shocks.[3] The general forms of the transmission curves are shown in the diagram. It will be seen that the curves



of the first and second "preliminary tremors," as Milne called the first two groups of waves, are curved, indicating that the velocity of transmission increases with the distance from the origin; a conclusion which had already been drawn from earlier, but less accurate, observations. Milne attempted to explain this by assuming that the path of the seismic disturbance lay along the chord and not along the earth's surface; this practically shortens the distance to the observing stations, and if the curves are plotted, with distances

[3] Rep. of the Com. on Seismol. Investig., B. A. A. S., 1902, p. 7.

measured along the chord, the curvature is considerably diminished; but later and more accurate observations show that even under this assumption the velocity still increases with the distance. The conclusion is unavoidable that as the path of the disturbance sinks deeper into the earth the velocity increases. The interior of the earth then is not a homogeneous but a refractive medium, and the path of the disturbance cannot be straight but must be curved with the concavity turned upward. This condition had been described by A. Schmidt as early as 1888.[4] Seismologists now believe that the three groups discovered by Oldham are respectively the longitudinal, the transverse and the surface waves. The transmission curve of the latter is a straight line indicating that the waves are transmitted with uniform velocity along the surface of the earth. They have affected seismographs after having passed completely around the earth. It cannot be said that the evidence, that the first two groups are respectively longitudinal and transverse, is complete; but it is sufficient, in connection with theory, to make seismologists fairly confident that the conclusion is correct; and the passage of transverse waves through the earth to great depths is proof that, to those depths, the earth is solid; for transverse waves cannot exist in a liquid. Further, since the velocity of transmission depends on the ratio of the elasticity to the density of the medium, and since both the longitudinal and transverse waves increase in velocity with the depth below the surface, both the elasticity of volume and the elasticity of figure of the earth, not only increase, but increase more rapidly than the density as we penetrate below the surface. The earth therefore is not only rigid, but its rigidity increases towards its center; though seismological evidence does not yet prove that this characteristic extends to the very center itself.

The next step was to determine the path of the waves in the earth and their velocity at different depths; the data for these determinations were the times of arrival of the earthquake waves at various distances from the origin; these times are collected in the transmission curves. At first sight this seems an insoluble problem;

[4] "Wellenbewegung und Erdbeben," *Jahreshefte für Vaterlands Naturkunde in Württemberg*, 1888, p. 248.

but, thanks to a remarkable mathematical theorem of Abel, it is not. It is clear that the time of arrival of an earthquake disturbance at a distant station will depend on the path followed and the velocity in different parts of the path, and if we make the reasonable assumption, which is borne out by observation, that the velocity is everywhere the same at the same depth, then it is evident, if the velocity increases continuously with the depth, that the transmission curves will be continuous without breaks, and their curvatures will nowhere make a sudden change. The mathematical solution of the problem has been obtained by Wiechert, Bateman and others, and concrete results have been obtained by Wiechert and his assistants, so that we now know the paths of the waves and their velocities with a fair degree of accuracy, at least to a considerable distance below the surface. But the questions arise: do the velocities increase continuously with the depth; and if so, how? questions which could be answered by the study of perfect transmission curves; but even imperfect curves yield some information; which, however, may be so faulty that it must be received with great caution. Milne, who has done such excellent pioneer work in seismology, was the first to propose and attempt to answer these questions.[5] He thought the transmission curve could be satisfied by supposing the earth to consist of a solid core having a radius of nineteen twentieths of the earth's radius, and surrounded by a thin shell. The core was of uniform density and elasticity, so that the velocity of propagation in it was uniform, and the paths of the rays would be straight lines. The velocity in the shell was much less than in the core. These conditions satisfied fairly well the very imperfect transmission curve of 1902, but they may be dismissed without further consideration, for such an earth could not satisfy the astronomic requirements, which exact, at the same time, the proper mean density and moment of inertia.

Benndorff in 1906 thought he found evidence of a central core of about four fifths the earth's radius, surrounded by two shells, the outer one having the same thickness as Milne's.[6] In the same

[5] Rep. of the Com. on Seismol. Investigation, B. A. A. S., 1903, p. 7.

[6] "Ueber die Art der Fortpflanzungsgeschwindigheit der Erdbebenwellen in Erdinnern," *Mitt. d. Erdbeben Com. k. Akad. Wiss. in Wien*, 1905, Nos. XXIX. and XXXI.

year Oldham deduced from the transmission curves a central core of not more than four tenths the earth's radius in which the velocity was distinctly less than in the surrounding shell.[7] Neither of these arrangements have been shown to conform to the astronomic requirements. Oldham's conclusions are based on what he considers a distinct break in the transmission curve of the transverse waves at distances between 120° and 150° from the origin; but when we remember that fully 95 per cent. of the energy of an earthquake shock comes to the surface within the hemisphere having the origin as its pole, we see that the data for great distances must be too imperfect to yield very reliable deductions.

Many years ago Roche showed that it was quite possible to determine a distribution of density in the earth which would be discontinuous at several levels, but which would still be astronomically satisfactory. Wiechert, in 1897,[8] showed that such a system might consist of a central core of radius about 4,900 km. or three fourths of the earth's radius, consisting of iron with a density of about 8.3, surrounded by a stony shell about 1,500 km. thick and with density varying from 3 to 3.4. It was natural that he should examine the transmission curves to see if they supported his ideas; and at the Hague meeting of the International Seismological Association in 1907 he announced that they did. At the Manchester meeting of the same association in 1911 he announced the existence of two shells around the central core. In 1914 Gutenberg (one of Wiechert's assistants) announced the existence of three shells.[9] In addition to ordinary times of transmission, Gutenberg also used the times of waves reflected at the earth's surface and the variations in the amplitude; it is evident that a wave which crosses the boundary of the core will experience reflection and refraction; and whichever part is later observed at the surface of the earth will have a dis-

[7] Constitution of the Interior of the Earth, *Quart. Jour. Geol. Soc.*, 1906, Vol. LXII., p. 456.

[8] "Ueber die Massenvertheilung im Innern der Erde," *Nachr. k. Gesells. Wiss. Göttingen*, 1897; *Math.-phys. Kl.*, p. 221.

[9] "Ueber Erdbebenwellen," VIIA. *Nach. k. Gesells. Wiss. Göttingen; Math.-phys. Kl.*, 1914, p. 1; references to the earlier numbers of the series are given in this paper.

tinctly smaller amplitude than the wave which just missed penetrating into the core. The following table shows the positions of the boundaries of the shells and of the core, and the velocities of the longitudinal waves *P* and of the transverse waves *S;* it will be noticed that it is only at the boundary of the central core that any marked sudden change in velocity occurs.

	Veloc. km./sec.	
Depth, kms.	*P.*	*S.*
0	7.17	4.01
1200	11.80	6.59
1700	12.22	6.86
2450	13.29	7.32
	13.15	7.20
2900	13.15	7.20
	8.50	4.72
6370	11.10	6.15

The remark regarding Oldham's results applies also here, namely that it is questionable whether the observations at distances greater than 100° or 120° are sufficiently accurate to justify such definite conclusions. Gutenberg had the advantage, however, of more accurate observations than Oldham, and also of measures of amplitudes. There is no *a priori* reason why the earth might not be made up of a number of shells, but there should be satisfactory evidence for any proposed system; and it must be shown to satisfy the astronomic requirements; or, at least, not to contradict them. Gutenberg's system does not correspond with Wiechert's system of 1897. In the latter a marked change in physical properties occurs at a depth of 1,500 km.; in the former, at a depth of 2,900 km.; and in crossing into the core, the ratio of the elasticity to the density, according to Gutenberg, rapidly loses six tenths of its value. This change might be the result of a great increase in density or a great decrease in elasticity; it may be questioned whether the former is compatible with the astronomic requirements, and whether the latter is compatible with the high rigidity which we know the earth, as a whole, has. So far no answer has been given to these questions.

In 1879 George and Horace Darwin attempted to determine the rigidity of the earth by measuring the deviation of the vertical under

the attraction of the moon. If the earth yielded like a fluid, its surface would always remain at right angles to the vertical, and a pendulum would remain relatively stationary for all positions of the moon; if the earth were absolutely rigid, the moon's attraction would deflect the pendulum an extremely small amount, but an amount capable of being measured. The Darwins did not obtain definite results because the disturbances of their pendulum were greater than the deflections they attempted to determine.

A little later von Rebeur-Paschwitz attacked the same problem with better success, using a horizontal pendulum.

Hecker, in Potsdam, and Orloff, in Dorpat, have repeated von Rebeur-Paschwitz's experiment; and both found values for the average rigidity of the earth comparable with that of steel. But, what was most remarkable and what is still unexplained, the rigidity was apparently greater in an east-west than in a north-south direction. Orloff, experimenting at a greater distance from the ocean, found a smaller difference than Hecker did, and it has been suggested that the tides of the ocean are the cause of the difference. The International Seismological Association, at its Manchester meeting in 1911, made plans to repeat the experiments in Paris, in central Canada, in the middle of Southern Africa and in the middle of Russia; but no reports have yet come from these stations.

In the autumn of 1913, Michelson attacked the same problem by a new method, which seems capable of yielding more accurate results than the horizontal pendulum. He measured the deflection of the vertical under the influence of the moon by what was practically a water level 500 feet long, sunk six feet in the earth.[10] Michelson's results for the E-W rigidity do not differ greatly from those of Orloff; but his N-S rigidity is somewhat less than Orloff's. Michelson's experiments also show that the viscosity of the earth must be as great as that of steel. These experiments are of great interest; they should be repeated at various places, and especially at places symmetrically situated with respect to the great oceans, and on mid-oceanic islands, in order to determine how far they are affected by the oceanic tides.

[10] "Preliminary Results of Measurements of the Rigidity of the Earth," *The Astrophysical Journal*, 1914, Vol. XXXIX., p. 97.

We can say in conclusion, that the transmission of transverse earthquake waves shows that the earth is solid, at least to a great depth below the surface; and that experiments on the deflection of the vertical show that it is quite as rigid and as viscous as steel. There are still difficulties in the interpretation of the observations, but their elucidation cannot alter the general character of the conclusions.

III.

THE EARTH FROM THE GEOPHYSICAL STANDPOINT.

By JOHN F. HAYFORD.

(*Read April 24, 1915.*)

This is a broad topic on which much intensive thinking has been done by many men. It is impossible to treat it adequately or comprehensively in the short time available.

In this address an attempt will be made to so concentrate attention on a certain few points as to tend to clarify existing ideas and to correlate them. An attempt will also be made to help in locating the lines of least resistance to future progress in the study of the earth.

The size of the earth, as well as its shape, is now known with such a high degree of accuracy that the errors are negligible in comparison with the errors in other parts of our knowledge of the earth. The probable error of the equatorial radius is less than 1/300000 part, and of the polar semi-diameter is about the same.

The three physical constants of the earth, and of its different parts, on which you are now asked to concentrate your attention are the density, the modulus of elasticity, and the strength.

It is important to know as much as possible about the density. The more one knows about the density in all parts of the earth the more surely and safely one may proceed in learning other things about the earth.

The modulus of elasticity at each point in the earth controls the behavior of the earth under relatively small applied forces.

The strength of the earth, at each point, as measured by the stress-difference at that point necessary to produce either slow continuous change of shape or rupture, decides the behavior of the earth under the greater forces applied to it.

As to density we know that the earth's surface density is about

2.7, that the density probably increases continuously with increase of depth, that the density at the center is probably about 11, that the mean density is about 5.6, and that within a film at the surface of a thickness of about one fiftieth of the radius of the earth there is isostatic compensation which is nearly complete and perfect as between areas of large extent.

The manner of distribution of the isostatic compensation with respect to depth, and the limiting depth to which it extends are but imperfectly known. Nevertheless it appears that above the depth, 122 kilometers, the compensation is nearly complete even though there may be some compensation extending beyond that depth.

Two general lines of evidence are available in determining the modulus of elasticity of the earth, that from earthquake waves, and that from earth tides.

There are many inherent and extreme difficulties in the way of securing reliable evidence as to the modulus of elasticity from earthquake waves.

To 1913 the accuracy of available observations of tides in the solid earth was insufficient to furnish a basis for reliable conclusions. Nevertheless the estimates of the modulus derived from these early observations were a fair approximation to that given by the very recent and much more accurate observations.

Dr. Michelson and those associated with him in the observation of earth tides at the Yerkes Observatory since 1913 have developed a method of observing which is of a new order of accuracy such that the minute changes of inclination at a given point due to earth tides may be determined with an error of less than one per cent.

These observations make the modulus of elasticity of the earth as a whole about like that of solid steel, namely (8.6) (10^{11} C.G.S.).

It is the modulus of elasticity of the earth as a whole which is measured in this case.

It is eminently desirable to determine if possible whether the modulus of elasticity varies with increase of depth. The Michelson apparatus possibly opens the way to such a determination. Suppose that the apparatus is used on the shore of the Bay of Fundy. Twice a day a large excess load of water is placed in the bay by the tidal oscillation and as frequently the water load is reduced below

normal. The stresses produced in the body of the earth by these changes of load applied over an area only about 30 miles wide are probably confined almost entirely to the first 100 miles of depth. The magnitude of changes of inclination produced at an observing station on the shore by the changing water load would, therefore, be dependent primarily on the modulus of elasticity of the material below and around the bay to a depth of less than 100 miles. The observations might serve, therefore, to determine a modulus of elasticity of the surface portion of the earth rather than of the whole earth.

Turn now to the third of the physical constants which it was proposed to examine, namely the strength.

Among the forces which we may consider as furnishing tests of strength are: (1) the forces involved in earthquakes, (2) the weight of continents, and (3) the weight of mountains.

The forces which produce the more intense earthquakes evidently cause stress-differences locally which are beyond the breaking strength of the material. However from earthquakes we may obtain but little information as to the strength of the earth material because the intensity of the stress-differences cannot be reliably determined. We know simply that the intensity exceeds the breaking strength of the material, at the points of rupture.

It is uncertain how great are the maximum stress-differences produced by the weight of continents. One great difficulty in computing these stress-differences arises from the fact that the isostatic compensation of continents, now known to exist, reduces the stress-differences much below what they would otherwise be. Love computed the maximum stress-differences thus reduced as .07 ton per square inch. Darwin computed the greatest stress-difference due to the weight of the continents, without isostatic compensation, as 4 tons per square inch. If each of these computations were based upon assumptions which correspond closely with the facts one should be warranted in drawing the conclusion that the maximum stress-difference caused by the actual continents supported in part by the actual isostastic compensation is between .07 and 4 tons per square inch, and that it is much nearer to the smaller than to the larger value. But a close examination of either of these computations shows that it is based

upon assumptions made to simplify and shorten the computations, which assumptions depart widely from the facts and tend strongly to make the computed stress-differences much smaller than the actual. For example, both Darwin and Love used in their computations hypothetical continents represented by regular mathematical forms in the place of the actual continents with their many irregularities. The maximum stress-difference caused by the actual continents is necessarily much greater than would be produced by the assumed smoothed out, regular, symmetrical continents.

Similarly, no adequate computations have been made to determine the maximum stress-difference due to the mountains. Darwin computed the maximum stress-difference produced by two parallel mountain ranges, of density 2.8, rising 13,000 feet above the intermediate valley bottom, to be 2.6 tons per square inch. Love, for the same mountain ranges, but with isostatic compensation taken into account, computed the maximum stress-difference to be 1.6 tons per square inch. In this case the computation indicates that the isostatic compensation reduced the maximum stress-difference to but little more than one half what it would otherwise be. Here again both the computed maximum stress-differences have been greatly reduced by substituting hypothetical smoothed-out mountains in the place of the actual irregular unsymmetrical mountains.

To the person who is trying to get a true picture of the present state of stress in the earth, two very important facts are made evident by a comparison of the Love and the Darwin computations. First, the existence of isostatic computation greatly reduces the stress-differences which would otherwise be produced by the weight of the continents and mountains. Second, the depth at which the maximum stress-difference tends to occur is evidently very much less with isostatic compensation than without it. These two conclusions, based on the differences between the two computations, are apparently reasonably safe even in spite of the same wild assumptions on which both the computations were based.

Note that even a little information as to the distribution of densities—a little information about isostatic compensation—profoundly modifies the conclusions as to the state of stress in the earth. It should, therefore, be clear why it was so emphatically stated in

an earlier part of this address that information as to the distribution of density in the earth is necessary in order to make safe progress in learning other things about the earth.

Is the earth competent to withstand without slow yielding the stress-differences due to the weight of continents and mountains, the isostatic compensations being considered? From the computations by Darwin and Love, considered in the light of the assumptions made by them to simplify the computations, I estimate that it is probable that the actual mountains and continents with all their irregularities of shape and elevation possibly produce stress-differences in some few places as great as four tons per square inch, and certainly produce stress-differences at many places as great as two tenths of a ton per square inch. The material would certainly yield slowly under such stress-differences especially when they persist continuously over long periods of time and throughout large regions. Four tons per inch is the breaking or rupture load for good granite, one of the strongest materials existing in the earth in large quantities. Two tenths of a ton per square inch is the safe working load used by engineers for good granite. There is abundant evidence from laboratory tests that the so-called yield point on which the engineer bases his estimate of safe working load for a given material is a function of the length of time the load is applied and the delicacy of the test. The longer the time of application and the more refined the test to determine the permanent yield the lower the observed yield point. In the case of the test in progress in the earth the time of application is indefinitely long and the test is extremely refined inasmuch as the minimum rate of yielding which may be detected is exceedingly small.

If an engineer wishes to know whether a bridge, or foundation, or building, or railroad rail is yielding under stress-differences which have been brought to bear upon it he looks for evidence of distress, for rivet heads popped off, scaling from the surface, settling, cracks, or even changes in microscopic structure. The geologists have made very extensive corresponding examinations of the earth. Everywhere they find evidence that the earth has yielded. On the one fourth of the earth's surface exposed to examination, the land, there is no part for which the evidence does not indicate

past uplift, or subsistence, or horizontal thrust, or cracking under tension, or cracking produced by shear, or microscopic yielding in detail such as produces schistosity for example, or some other form of past yielding to stress-differences. The physicist studying the earth must take this overwhelming mass of evidence into account and must conclude that the earth habitually yields slowly to the stress-differences brought to bear upon it. Please note that I do not assert that the stress-differences are all due to gravity.

I propose now to state what are in my opinion probably the lines of least resistance to future progress in studying the earth from the physical standpoint. I propose to outline what I believe to be the most effective methods of attack, and to indicate some of the conclusions which will probably be reached. I am led to this procedure by two considerations. First, I find it possible to state certain of my opinions as to the net outcome of past investigations most clearly in that form—and time presses. Second, I indulge the hope that such an outline which is frankly an expression of judgment based on evidence much too weak and conflicting to be proof, may possibly kindle the imagination of some man or men, and so lead to vigorous attacks upon the problem and to future progress.

In attacking the problems of the earth one should assume at the outset that the phenomena exhibited are very complicated, that they are probably due to various simultaneous actions, and that the various actions are probably closely interlocked, modifying each other, though some are probably primary in importance and others secondary. Hence the most effective method of attack is probably one which includes a general correlation of apparently widely separated ideas and facts gathered from physicists, engineers, geologists, chemists, etc., and at the same time includes intensive attacks in detail on one after the other of single features of the problems which arise and an intensive working out of the possible consequences of said features.

It should be recognized at the outset that no observed behavior of the earth clearly warrants the assumption that the material of which it is composed differs radically in any way from that accesible at the surface. It should be assumed, therefore, that throughout the earth the materials are a mixture differing from the mixture

found at the surface only as the extreme pressure and temperature conditions at great depths directly and indirectly produce differences.

It should be kept clearly in mind that the geodetic evidence from observations of the direction and intensity of gravity indicates simply the present location of attracting masses, the present distribution of density. It furnishes no direct evidence whatever as to past distributions of density, or as to changes in density now in progress. But an understanding of the present distribution of density within the earth, especially near the surface, is so necessary to a true understanding of the present state of stress and of viscous flow in the earth that an understanding of the geodetic evidence is fundamental to progress.

Computations should be made in extension of those which have been made by Darwin and Love. The new computations should, however, deal with the actual irregular continents and mountains, not with regular substitutes. The computations should also take into account the bulk modulus of the materials composing the earth, that is these materials should be assumed to be compressible. Such computations will no doubt be both difficult and long. I believe that even a moderately vigorous attack along this line will show conclusively that the earth does not behave as an elastic body under the large loads superimposed upon it by the continents and mountains. I believe that the computed stress-differences will be found to be so large that the computation will be essentially a proof of viscous yielding.

Next make the contrasting assumption that the material composing the earth is competent to withstand but little shearing stress, and that the pressure at any point is that due to gravitation acting on the mass in the column extending from the point vertically to the surface. Let it be assumed that isostatic compensation exists, is uniformly distributed with respect to depth, and is complete at depth 122 kilometers. Consider the actual topography and form a mental picture as accurately as possible of the viscous flows which would take place on the assumption that at each level the material would flow horizontally from regions of greater pressure to regions of less pressure along lines of maximum rate of change of pressure, and that the time rate of such viscous flows would tend to be pro-

portional to the space rate of change of pressure. The flows would all be found to be away from beneath high regions toward low regions, from continents toward oceans, from mountains toward valleys.

After such a picture has been clearly formed assume that the isostatic condition is disturbed by long-continued erosion and deposition producing changes in the surface elevations and surface loads. On the same assumptions as to the nature of the viscous flows as before, form a new picture of the viscous flows which would now be in progress. It will be found that under the new conditions the viscous flows near the surface would still be away from high areas and toward low areas, but in general they would be slower than before. At greater depths, however, it will be found that the viscous flows would be undertows from regions of recent deposition toward regions of recent erosion. These undertow flows would in general tend to be in the direction opposite to recent surface transportation of material. This picture would serve as a first approximation to an understanding of the mechanism of isostatic readjustment. The undertows would be found on these assumptions to extend to a considerable depth, certainly more than 122 kilometers.

Next one should picture the changes in density which would be produced by the viscous flows. The density should be pictured as decreasing in regions from which material is being carried away by the flow and increasing in regions to which the material is being carried. It will be noticed as soon as such a picture is formed that every undertow flow at any level tends to equalize pressures at lower levels. This will have a strong tendency to make the prevailing undertows occur at much higher levels than they otherwise would.

Let it be assumed that the viscous material offers some small resistance to shear and still has elastic properties to a slight degree. The condition assumed originally that the pressure at a point depends simply upon the weight of the material above that point will be disturbed thereby. Form as clear a conception as possible of these disturbances and the modifications of the flows produced by them. I believe the modifications will be found to be important, and that they will be found to be such as tend to confine the effects of surface changes of load to a depth which is a small fraction of the radius.

So much for the direct effects of gravity which it seems important to picture clearly. Next study other effects, some of which are indirectly produced by gravity.

First study the modifying effects of changes of temperature. Wherever viscous flow takes place in the quasi-solid portions of the earth there heat is necessarily developed in amount equivalent to the mechanical energy expended in overcoming the resistance to flow. This will tend to increase the volume of the material, to increase the pressure, and to raise the surface above the region of viscous flow. It is probable also that the increase of temperature will tend to weaken the material, thus emphasizing the weakening produced by the damaging mechanical effects of the flow.

This temperature effect is probably locally important.

Beneath areas of recent deposition the temperature of a given part of the buried material will slowly increase for long periods of time, on account of heat conducted up from below and prevented by the new blanket of deposited material from rising to the surface so freely as before. Conversely, beneath the areas of recent erosion the temperature of a given portion of material will decrease. The ultimate limit of change will tend to be in each case not greater than about one degree Centigrade for each thirty-two meters of depth of erosion or deposition. These temperature changes tend ultimately to lower areas of recent erosion and to raise areas of recent deposition, possibly as much as one thirtieth of the thickness of the erosion or deposition,—the temperature effect taking place much later than the erosion or deposition which initiated it.

Study next the effects which may be computed from the bulk modulus of elasticity. Beneath areas of erosion a given particle of matter tends to rise by an amount which may be computed from the bulk modulus of material, and similarly a particle tends to fall beneath an area of deposition. If the depth to which the elastic phenomena extend is as great as 122 kilometers and the bulk modulus is 500,000 kilograms per square centimeter (corresponding to granite) the rise or fall of a particle near the surface will tend to be at least 1/50th part as great as the thickness of the material eroded or deposited. This is a change so large as to have considerable effects in modifying or magnifying the actions which would

otherwise occur. Possibly this elastic change is much larger than the estimate here given. Of course if the erosion or deposition takes place in a small area only, such elastic response will be largely inhibited by surrounding material on which the load has not been directly changed. But under large areas of erosion or deposition such action must take place and extend to depths possibly as great as 122 kilometers.

Study next the modifying effects, on the phenomena already pictured, of chemical changes which are probably produced in the earth by changes of pressure. The expression "chemical changes" is here used in the broadest possible sense. A relief of pressure at any given point in the earth necessarily favors such chemical changes as are accompanied by increase in volume and reduction of density. Increase of pressure tends to have the reverse effect. Such changes tend to reinforce and extend in time the effects just referred to which may be computed from the bulk modulus of elasticity. It is important to estimate such changes as well as possible from all available evidence, such for example as that furnished by chemists, by geologists, and by such investigations of rock formation as have been conducted at the geophysical laboratory in Washington. I believe the possible effects of this kind will be found to be so large as to be of primary importance.

Evidence has accumulated during the past few years which makes it reasonably certain that with increased pressure, as at the great depths in the earth, the rigidity and the viscosity of the material also necessarily increase. This tends to cause the viscous flows to take place at higher levels than they otherwise would. This should be taken into account.

Next a reëxamination of the conceptions so far formed should be made to ascertain to what extent and how they would be modified if one started with some other reasonable assumption as to the limiting depth of present isostatic compensation or some other reasonable assumption as to the law of distribution of the compensation with regard to depth.

Next full and extensive comparisons should be made between the hypothetical phenomena on the one hand pictured as made up primarily of viscous flows, modified by some elastic effects, ini-

tiated in part by surface transfers of load, modified by changes of temperature, modified by chemical changes and in the other ways, and on the other hand the facts of the past as to the behavior of the earth recorded in the rocks and read by geologists and others. This comparison should be used to the fullest possible extent to evaluate the relative importance of the various elements in the actions.

In making this comparison of various hypothetical phenomena with the great accumulated mass of geological facts it should be recognized at once that it is false logic to reason that if a given hypothesis does not account for all the observed facts the hypothesis is necessarily erroneous. On the contrary it is true logic in dealing with such a problem as the earth seen from a physical standpoint to reason that the more facts are accounted for by a given hypothesis the more certain it is that said hypothesis is a statement of a controlling element in the complex phenomena and then to study the facts which appear neutral, or conflicting, with reference to the hypothesis, considering them as indicators of other elements of the phenomena which one should attempt to embody in other supplementary hypotheses.

I submit that in studying the earth it is a mistake to think that there is any necessary conflict between the idea that the earth behaves as an elastic body and the idea that it is yielding in a viscous manner. A body may behave in both ways at once. The earth is probably acting largely as an elastic body under small forces which change rapidly and at the same time is yielding in a viscous manner to forces of larger intensity which are applied in one sense continuously for long periods.

The object of this address will have been accomplished if it serves in time to arouse the imagination and interest of some one and to guide him to greater effectiveness in attacking the problems presented by the earth as seen from the geophysical standpoint.

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MORPHOLOGY AND DEVELOPMENT OF AGARICUS RODMANI

(PLATES VII.–XIII.)

BY GEO. F. ATKINSON.

(*Read April 23, 1915.*)

INTRODUCTION.

Agaricus rodmani[1] was described by Peck in 1885, from specimens growing in "grassy ground and paved gutters" at Astoria, Long Island. As to its habitat and occurrence a more specific statement is made in 1897, in that it "grows in grassy ground and even in crevices of unused pavements and paved gutters in cities,"[2] from May to July, and is said to be rare. It has been observed in the city of Ithaça, N. Y., for a number of years, where it is usually found growing in the parking between the sidewalks and street curbing, or even in the crevices of stone paved streets and gutters, and also in grassy ground along the street railway or along walks on the border of groves. The material for this study was collected in August, 1914, along the Ithaca street railway and by the side of paths along the border of groves on the campus. In these places the mycelium in spots was often very abundant so that lumps of soil resembling a fine quality of spawn were exposed in digging for the young stages. The young fruit bodies collected were scattered on these cords of mycelium, the material and conditions offering very clear evidence of the normal development of the basidiocarps. The material was fixed in chrom-acetic fluid and sectioned in paraffin.

The features of interest in the morphology and development of *Agaricus rodmani* which I have considered in the present study are as follows: (1) the duplex character of the annulus, or ring, on the stem, and its significance; (2) the origin of the hymenophore funda-

[1] N. Y. State Mus. Nat. Hist. Rept., **36**, 45, 1885.
[2] N. Y. State Mus. Nat. Hist. Rept., **48**, 139, 1897.

ment; (3) the differentiation of parts in the primordial ground tissue; and (4) the origin and development of the lamellæ. The peculiar form and position of the annulus on the stem has suggested a resemblance to a volva, a structure not admitted in the genus *Agaricus* as now limited; while the subject of the origin and development of the lamellæ has acquired new interest in all of the Agaricaceæ since the accuracy of observations and the correctness of the statements covering a period of more than a half a century, in regard to this topic, have recently been called in question. Without further preliminary remarks we may proceed to an account of the present investigation, and to a consideration of the various matters involved.

I. The Duplex Annulus and its Significance.

The Annulus.—The annulus is situated near the middle of the short stem, or even near its base. It is usually very thick next the stem and is divided into an upper and lower limb by a deep marginal groove as is clearly seen in the photographs reproduced in Plate I. In those cases where the annulus is near the base of the stem, Peck was impressed by its suggestion of "the idea of a volva" (*l. c.*, 45). Before the expansion of the pileus, while the veil is still attached to the stem and pileus margin, a longitudinal section of the plant shows very clearly that the lower limb of the annulus lies on the outer (upper) side of the pileus margin (see Plate VII., upper right hand and lower left hand figures). The marginal veil is very thick and the epinastic growth of the pileus margin crowds the latter into the veil tissue and against the stem. The position of the lower limb of the annulus therefore corresponds to that of the volva limb of the *Amanitas*.

The plates represented in the upper group of Plate VII., were collected on the Cornell University campus, those in the upper group during August, 1911, along a path in the edge of a small wood not far from the street; those in the lower group, July, 1913, along the street railway and parking by East Avenue. In the expanded specimens, the pileus ranged from 6 cm. to 8 cm. in diameter. The plants were smaller than those represented in Plate VIII., but since they were abundant and in all stages of development they present in

an excellent way the different details of the veil and annulus during expansion of the plant. Those represented in Plate VIII., were collected by Mr. Wood, June 28, 1915, in the parking between the sidewalk and street, on Stewart Avenue, in front of the Town and Gown Club, Ithaca, N. Y. They were very robust specimens, and show the great distance between the upper and lower limb of the annulus. They are reproduced here real size.

A thin outer layer of the lower limb of the annulus is continuous below with the outer layer of the stem, and also with a very thin surface layer of the pileus. As the stem elongates at the time of the expansion of the plant, this outer layer of the stem lags behind and is thus torn into irregular patches shown very clearly in the two upper left-hand figures of Plate VII. The edges of these patches are frequently warped away from the stem, thus showing a tendency to exfoliation. This is especially marked in the case of the surface layer of the stem next the lower limb of the annulus. The warping upward of this layer, after it has been severed from its connection below, often gives the appearance of a double edge to the lower limb of the annulus, as shown in the lower right-hand figure of Plate VII., where the upper limb of the annulus has not yet broken away from the pileus margin.

The very thin layer on the pileus which is also continuous with a thin outer layer of the lower limb of the annulus often shows a tendency to exfoliation. This partial exfoliation of the stem and pileus surface is clearly marked where the basidiocarps are somewhat soiled by contact with particles of earth, as they are likely to be during the period of subterranean growth.

The outer portion of the lower limb of the annulus, as well as the corresponding thin, and partially exfoliating surface layer of the pileus and stem are derived from the outer layer of the blematogen. The blematogen layer, as I have interpreted it, is present in the genus *Agaricus* as well as in *Amanita*. In the species of *Amanita* thus far studied,[3] the blematogen at length is clearly separated from the pileus by a cleavage layer, arising from the gelatinization, or other kind of disintegration, of the external layer of the pileus primordium, thus

[3] Atkinson, Geo. F., "The Development of *Amanitopsis vaginata*," *Ann. Myc.*, 12, 369–392, pls. 17–19, 1914.

giving rise to the teleoblem, or finished volva. But in the genus *Agaricus*[4] no such cleavage layer is formed, and the surface of the pileus primordium becomes consolidated with the blematogen layer which here does not form a true volva, or teleoblem.

The lower limb of the annulus of *Agaricus rodmani* is not, therefore, strictly homologous with the volva of the *Amanitas,* not even including the thin layer of the stem and pileus which sometimes tends to peel off, since it does not comprise all of the blematogen layer, nor is it separated from the pileus by a distinct cleavage layer. If it were homologous with the volva of the *Amanitas,* then this species would represent a generic type distinct from *Agaricus* (*Psalliota*). In fact other species of *Agaricus* frequently show a similar condition of the annulus, *i. e.,* where the margin is "grooved," due to the inset of the pileus margin into the veil where the conditions for the robust development of the veil are favorable. In *Agaricus campestris* the annulus frequenlty presents a grooved margin, not only in the case of cultivated forms, but more rarely in the feral state. This condition is well shown in Plates 11 and 12 of my article on *Agaricus campestris.*[5] In Fig. 20 of that article the lower limb of the annulus has broken away from the outer surface of the incurved pileus margin, while the upper limb is still attached to the edge of the pileus. In Figs. 18 and 19 the upper limb has also become freed from the pileus margin and the grooved character of the edge of the annulus is very distinctly shown. In Fig. 15 of the same article, sections of the young basidiocarps show very clearly the position of the lower limb of the annulus extending over the outer (upper) side of the pileus margin. Fig. 20 also shows very clearly that the annulus as a whole is ripped off from the lower part of the stem, being an exaggerated case of the slight peeling up of the thin surface layer of the stem mentioned above in *Agaricus rodmani.* That the

[4] Atkinson, Geo. F., "The Development of *Agaricus arvensis* and *A. comtulus*," *Am. Jour. Bot.*, 1, 3–22, pls. 1, 2, 1914.

Atkinson, Geo. F., "Homology of the Universal Veil in *Agaricus*," *Myc. Centralb.*, 5, 13–19, pls. 1–3, 1914.

Atkinson, Geo. F., "The Development of *Lepiota clypeolaria*, *Ann. Myc.*, 12, 346–356, pls. 13–16, 1914.

[5] Atkinson, Geo. F., "The Development of *Agaricus campestris*," *Bot. Gaz.*, 42: 241–264, pls. 7–12, 1906.

lower limb of the annulus in *A. rodmani* is merely a part of the marginal veil is clearly seen in the sectioned plants shown in the lower groups of Plate VII., where the connecting portion between the two limbs is clearly differentiated from the surface of the stem with which it is in contact, a situation very different from that in *Amanita* where the volva has no such relation to the annulus.

Comparison of Agaricus rodmani with other Species of Agaricus. —This extensive peeling, or ripping upward of the annulus from the lower part of the stem in *Agaricus campestris* is the cause of the more extensive, *i. e.*, broader, veil and annulus than is characteristic for *Agaricus rodmani.* Peck regards this species as intermediate between *Agaricus campestris* and *A. arvensis,*[6] resembling the former in size, shape and general appearance; the latter in the "whitish primary color of the lamellæ," in the occasional yellowish tints of the pileus, and the occasional rimose under surface of the annulus. The robust character of the annulus of *Agaricus rodmani* and the thick flesh of the pileus margin crowded by epinastic growth against the stem deepens and widens the groove on the edge of the annulus. This, together with the very short stem, in comparison with the longer stem of *Agaricus campestris* and *A. arvensis,* is, I think, largely responsible for certain differences in the character of the under surface of the annulus in the different species. In the species with the longer stem more stretching of the stem occurs and the annulus (or veil) is ripped upward from a greater extent of the stem surface. The radiately grooved character of the under surface of the annulus, in certain species (*A. arvensis* Schultz, *A. abruptibulbus* Pk., *A. placomyces* Pk., *A. hæmorrhoidarius* Schultz), or the coarsely floccose or scaly character in certain others (*Agaricus subrufescens* Pk., *A. augustus* Fr., or both features contained in some) is largely due to the fact that this part of the annulus is stripped from the stem and then brought under greater tension than the upper surface as the expansion of the pileus stretches the veil outward. All things considered *Agaricus rodmani* is much more closely related to *Agaricus campestris* than to any other of the species. It is very probably identical with *Agaricus campestris* var.

[6] N. Y. State Mus. Nat. Hist. Rept., 36, 45, 1885.

edulis Vitt.,[7] as I have elsewhere suggested[8] (1900, 1901, 1903, p. 20). Excellent figures of this variety are given by Vittadini (*l. c.*, pl. 6) and by Bresadola[9] (pl. 54).

II. Origin of the Hymenophore Primoridum

Primordium of the Basidiocarp.—The primordia of the basidiocarps are elliptical or oval in outline, and reach a diameter of 3 mm. or 4 mm. before there is any internal evidence of a differentiation of parts. The length is usually somewhat greater than the transverse diameter. In specimens not so well nourished differentiation may begin before the primordia have reached this size. The primordium, from the size of 2 mm. to 4 mm. in diameter, consists of a homogeneous interlacing of stout mycelial threads with rather thick walls. In primordia 3 mm. to 4 mm. in diameter the hyphae average about 5 μ to 7 μ in thickness, occasionally stouter ones are seen which measure up to 10 μ. More slender threads are also intermingled, but all sizes are so indiscriminately interwoven that no structural differentiation is perceptible. In smaller primordia the hyphæ average less in diameter. In most of the primordia examined, the sections are evenly stained throughout, but in a few a narrow zone a short distance from the surface stains more deeply than the external and internal tissue (Fig. 2). This suggested the possibility of a differentiation of an outer zone distinct from the bulk of the fruit body, which is sometimes present in *Agaricus campestris* and which I have called the *protoblem.*[10] A similar zone is found in some of the basidiocarps after the origin of the hymenophore fundament, but in the material which I have examined it is the exception rather than the rule, and I am inclined to the belief that it is due to some condition which affects the rate of growth or increase of cer-

[7] Vittadini, C., "Funghi Mangerecci," 44, 1835.

[8] Atkinson, Geo. F., "Studies of American Fungi; Mushrooms, Edible, Poisonous, etc.," 1st edition, I–VI., 1–275, 76 plates (223 figs.), Ithaca, N Y., 1900. *Idem,* 2d edition, I–VI., 1–322, 86 plates (250 figs.), Ithaca, N. Y. 1901. *Idem,* New York City, 1903.

[9] Bresadola, G., "Funghi Mangerecci e Velenosi," 1899.

[10] Atkinson, Geo. F., "The Development of *Agaricus arvensis* and *A. comtulus*," *Am. Jour. Bot.*, 1, 3–22, pls. 1, 2, 1914. "Homology of the Universal Veil in *Agaricus*," *Myc. Centralb.*, 5, 13–19, pls. 1–3, 1914.

tain individuals. A protoblem[11] is very likely present, but it is difficult to distinguish in primordia havng a subterranean origin because of the ease with which the delicate protoblem is removed while removing the soil, and especially in the forms and species of *Agaricus* with a white pileus. In those with a brown pileus, like *Agaricus campestris* var. *bohemia* of the commercial spawn growers, the delicate, white protoblem is very distinct.

Differentiation of an Internal Annular Hymenophore Primordium.—The first evidence of internal differentiation is the appearance of an internal annular zone of new growth in the region of the smaller end of the oval fruit body. This can be studied with advantage by means of serial, longitudinal sections. A median longitudinal section is shown in Fig. 3, while a "tangential" section, *i. e.*, parallel with the axis of the basidiocarp, but through one side of the annular zone of new growth is shown in Fig. 4. Diagrams 1 and 2 (in the text) show how the sections were made. Fig. 3 is from the region marked by the line 2, while Fig. 4 is from that marked by the lines 1 and 3. The darker staining areas in Figs. 3 and 4 mark the position of the zone of new growth. In the median

[11] The delicate, floccose, primary universal veil, or protoblem was observed by Fries on *Agaricus campestris* and a few other species, and called by him a subuniversal veil. Vittadini (in Fung. Mang., 147, pl. 18, fig. 2, 1835) describes and figures it in connection with his study of the development of his *Agaricus exquisitus*. But in this species he seems to confuse this delicate universal veil (protoblem) with what he terms the volva in several species of *Agaricus*. He also applies the term volva to the lower limb of the annulus in *Agaricus exquisitus* and in *Agaricus edulis*. He says (*l. c.*, 148) this delicate universal veil in *A. exquisitus* is perfectly similar to that which constitutes the veil of the "Tignose," *i. e.*, the scaly Amanitas like *A. muscaria*, etc. Vittadini also states (*l. c.* 147) that Trattinnick observed this delicate universal veil (protoblem) on *Agaricus edulis* (the species which Trattinnick describes as *A. edulis* is different from *A. campestris edulis* Vitt. or *A. rodmani* Pk.), but it appears that Vittadini misinterpreted Trattinnick's statment. The latter says, in order to prevent confusion one should avoid (*l. c.*, p. 73) taking for the edible one a mushroom (74), which may have also only the slightest trace of a membrane which in youth envelopes the entire mushroom, including pileus and stem, down to the roots. "Um Verwechslungen zu vermeiden, hüte man sich statt der Gugemuke einen Schwamm zu nehmen" (73), "(*d*) der auch nur die geringste Spur von einer Wulsthaut haben sollte, die in der Jugend den ganzen Schwamm mit sammt den Strunk und Hut bis auf die Wurzel verhüllet" (74 Die essbare Schwämme, 1830).

longitudinal section two such areas are seen, symmetrically situated on either side of the long axis and some distance from the surface of the fruit body. The annular zone is of quite limited extent as the

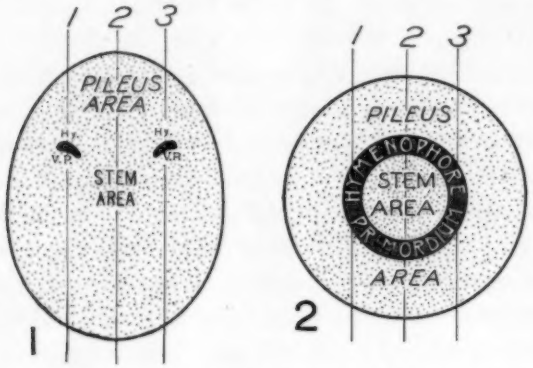


DIAGRAM 1 Lateral view through young basidiocarp representing early stage of differentiation into the primordia of the four principal parts; pileus area, stem area, hymenophore fundament (*Hy*) and veil primordium (*V. P.*).

DIAGRAM 2. Zenith view in young basidiocarp at same stage of fundaments, and annular hymenophore primordium. See text for details.

small area presented by its transection in Fig. 3 shows. The outline of this area in transection is somewhat elongated and rises at an oblique angle from the stem area, well shown in Fig. 3 and indicated in diagram 1. The area of the primordial hymenophore seen in the tangential section is much more extensive as shown in Fig. 4. The difference in the extent of these areas shown in median (Fig. 3) and tangential (Fig. 4) sections is clearly appreciated by reference to diagram 2.

Structure of the Young Hymenophore Primordium.—This internal annular zone of new growth arises by the origin of numerous slender hyphal branches, rich in protoplasm, which are directed downward, or obliquely downward and outward. They have a more direct course than the hyphae of the basidiocarp primordium, the latter irregularly sinuous and interwoven, while the hyphae of the young hymenophore primordium are nearly or quite straight. Because of their small diameter and their slender, gradually tapering ends, they easily crowd their way through the rather open weft of hyphæ forming the ground tissue or fundamental plectenchyma. Fig. 9 is a highly magnified view of the hymenophore primordium

shown in the section represented in Fig. 3, from the right-hand area. The dark area in Fig. 9 represents the mass of deeply stained hyphæ of the new growth zone. Because of the compactness of the tissue, very little detail is shown. But along the middle portion of the figure between the lighter, open mesh of the ground tissue below and to the right, and the dark area of the hymenophore primordium above and to the left, a number of hyphæ in advance of the others are shown extending into the loose mesh of the ground tissue. These are nearly parallel and their extremities are more or less distant, because they are in advance of the greater number of new branches present in the more deeply staining area. No annular gill cavity is present at this time.

Growth and Increase of the Hymenophore Primordium.—The growth and further organization of the hymenophore primordium is readily studied by the aid of similar serial sections of successively older stages of the basidiocarps. Sections of such stages are repre-

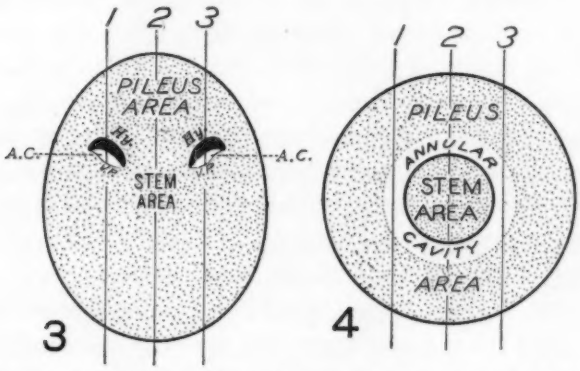


DIAGRAM 3. Lateral view through young basidiocarp at a slightly later stage of development than in diagram 1. *Hy* = hymenophore; *A. C.* = annular cavity; *V. P.* = veil primordium.

DIAGRAM 4. Zenith view in young basidiocarp at same stage of development. See text for details.

sented in Figs. 5–8 and 10–16. Diagrams 3 and 4 indicate how the sections were made. From the condition show in Figs. 3, 4 and 9, there is a rapid increase in the number of hyphæ in the zone of new growth, extending in the same direction, *i. e.*, downward and obliquely outward. During the increase in number the hyphæ become more crowded, are straighter and lie more nearly parallel. The

upper outer portion of this new zone of growth, *i. e.*, the hymenophore primordium, represents the early stage of the organization of the pileus margin: in other words, the annular internal zone of new growth is to be interpreted as the young primordium of hymenophore and pileus margin, the latter including the area from which the new hyphal branches arise as well as the basal area of these branches. Not only is there interstitial growth in the increase of these hyphal branches, the new ones crowding in between the older ones forming a more compact zone, but there is also a centrifugal increase in the periphery of the annular zone. The centrifugal growth of the pileus margin and hymenophore primordium is very characteristic.

The position and direction of the hyphæ of the young hymenophore primordium, as well as the increasing density of this area, is well shown in Figs. 10–16. The stem axis of all the figures is parallel with the long axis of the Plate. Several of these figures are highly magnified views of the hymenophore primordium shown in Figs. 5–7; Figs. 10 and 15 being highly magnified views of the hymenophore of Figs. 5 and 6, while Figs. 12 and 16 are highly magnified views of that in Figs. 7 and 8. Figs. 10 to 14 are from median longitudinal sections of the basidiocarps. Fig. 10 is from the right-hand side of the stem axis, *i. e.*, the stem axis is at the left. Figs. 11–14 are from the left-hand side of the stem axis, the stem axis therefore being on the right-hand of the figures. The increasing density of the elements of the young hymenophore is progressively shown in Figs. 10 to 13. With the increasing density the ends of the hyphæ reach more and more to the same level and thus tend to form an even surface which forms the transition to the palisade layer.

Origin of the General Annular Gill Cavity.—A striking feature in all these radial transections of the hymenophore zone and pileus margin is the curved outline of the zone as seen in transection. This is remarkably strong in Figs. 11 and 12 because the young hymenophore primordium extends for a considerable distance down around the apex of the stem fundament. This arched form of the young annular hymenophore zone is the result of epinastic growth of the pileus margin, which is very marked even in this very early stage in the organization. The rapid increase in the number of

the hyphæ in the young hymenophore, crowding in between the older ones, as well as their increase in diameter, produces a great pressure in this region. As a result of this increasing pressure within the arch a strong tension is exerted on the ground tissue below and adjacent to the arch. The ground tissue at this point is thus torn apart, forming a distinct opening, or cavity, beneath the young hymenophore, which is known as the annular gill cavity. The continuity as a general, annular, internal cavity can easily be determined by serial longitudinal sections through the young fruit body, the sections being made as indicated in diagrams 3 and 4, the knife travelling through the basidiocarp in the direction indicated by the lines 1, 2, 3. As the knife passes the region marked by the line 1, the sections will show a single cavity elongated transversely as shown in Figs. 6 and 8, 15 and 16. As the knife passes into the stem area the sections will show two cavities situated symmetrically as in Figs. 5 and 7 (or as in diagrams 3 and 4). Then as the knife passes out of the stem area, into the region indicated by the line 3, the sections will again show a single cavity elongated transversely.

The annular gill cavity[12] varies in strength in different individuals and at different stages of development. Sometimes it is very weak, at other times it is quite strong. The tearing apart of the ground tissue often leaves it with quite an open mesh, and the surface next the gill cavity is more or less frazzled. The gill cavity is stronger next the stem where the hymenophore is older, and is weaker toward the margin. Where the cavity is weak, isolated threads or irregular strands of the ground tissue are not completely torn away from the hymenophore, and the cavity is thus often traversed by lagging elements of the ground tissue. At a later stage, after the origin of the lamellæ, the annular cavity in some indi-

[12] In a recent paper, after describing the gills in *Coprinus micaceus*, Levine ("The Origin and Development of the Lamellæ in *Coprinus micaceus*," *Am. Jour. Bot.*, 1, 343–356, pls. 39, 40, 1914), makes the statement (p. 352) that "There is no general gill cavity as described by Hoffmann, deBary, Atkinson, and others." Since deBary ("Morphologie und Physiologie der Pilze, Flechten und Myxomyceten," 69, 1866) is the only person hitherto who has announced the presence of a general annular gill cavity in *Coprinus micaceus*, this statement by Levine can only be interpreted as a general denial of the presence of a general annular gill cavity in the species in which it has thus far been described, a rather rash statement which will be referred to again in the discussion of the origin of the lamellæ.

viduals may become nearly or quite closed by the increase in the elements of this ground tissue, which forms a portion of the marginal veil, but chiefly by the epinastic growth of the pileus margin which crowds this ground tissue up against the margin of the lamellæ, as shown in Figs. 32–38.

Organization of the Palisade Layer.—The level palisade layer of the hymenophore follows the primordial stage, immediately after the latter stage has become dense and compact by the increase in number and thickness of the parallel hyphal elements. The growing compactness of the primordial hymenophore zone is accompanied by the evening up of the hyphal ends into a plane surface. As the ends of the hyphæ broaden the free surface of the hymenophore becomes compact and smooth, or even. This is the level palisade stage of the hymenophore. It is a gradual, not abrupt, transition from the primordial stage. It begins next the stem, or in many cases on the outer surface of the upper part of the stem fundament as shown in Fig. 12. Here the palisade area, in radial section, rises upward at a strong oblique angle from the axis of the stem, and then grades into the primordial area toward the left. The palisade area progresses, like the primordial area and the pileus margin, in a centrifugal direction, the older portion lying next to, or on the upper part of the stem fundament.

The level palisade layer of the hymenophore, preceding the origin of the lamellæ, was first described by Hoffmann[13] in 1856, 1860, and 1861, in about a dozen species (see the later paragraph on the origin of the lamellæ for a list of species). DeBary[14] (1859, p. 386, 394) described the palisade layer of the young hymenophore in *Nyctalis asterophora* and *parasitica,* as having radial folds from its

[13] Hoffmann, H., "Die Pollinarien und Spermatien von Agaricus," *Bot. Zeit.,* **14**: 137–148; 153–163, pl. 5, 1856. Beiträge zur Entwickelungsgeschichte und Anatomie der Agaricinen," *Bot. Zeit.,* **18**: 389–395; 397–404, pls. 13, 14, 1860. Icones Analyticae Fungorum; Abbildungen und Beschreibungen von Pilzen mit besonderer Rucksicht auf Anatomie und Entwickelungsgeschichte," 1–105, pls. 1–24, 1861.

[14] DeBary, A., "Zur Kenntnis einer Agaricinen," *Bot. Zeit.,* **17**: 385–388; 393–398; 401–404, pl. 13, 1859.

[15] DeBary; A., "Morphologie und Physiologie der Pilze, Flechten und Myxomyceten," Leipzig, 1866. "Vergleichende Morphologie und Biologie der Pilze, Mycetozoen und Bacterien," 1884. "Comparative Morphology and Biology of the Fungi, Mycetozoa and Bacteria," Oxford, 1887.

earliest appearance. But as this interpretation was shown by Hoffman (1860, p. 402) to be wrong, deBary[15] (1866, p. 63; 1884, p. 58, 312; 1887, p. 55, 289) studied a number of other forms and agreed with Hoffman that the earliest stage of the young palisade hymenophore was level, or smooth.

III. The Differentiation of Parts in the Primordial Ground Tissue.

There are four principal parts of the fruit body which are differentiated in the ground tissue of the basidiocarp primordium, the *hymenophore, pileus, stem* and *veil*. The primary differentiation in the ground tissue of *Agaricus rodmani* is the origin of the hymenophore primordium. As described above this arises as an internal annular zone of new growth, a little above the middle of the small oval primordial basidiocarp. It consists of numerous hyphal branches which extend downward and obliquely outward. These new hyphæ are nearly or quite parallel, are at first slender and taper very gradually to the free end. This form assists them in making their way through the mesh of the ground tissue. They are rich in protoplasm, become compacted by increase in number and diameter, and thus in sections, take on a deep color when stains are applied (see Figs. 3–16). The origin of this internal hymenophore zone differentiates at once the stem and pileus areas, or fundaments, but the organization of the stem and pileus occurs later.

In the early origin of the primordial hymenophore zone, *Agaricus rodmani* agrees with *Agaricus campestris*[16] as presented in a study of the commercial varieties, *alaska* and *bohemia*. In that paper I pointed out that we should not necessarily expect the first evidence of differentiation to be the appearance of the hymenophore primordium in plants not yet studied though it is probable that at least some of the other species of *Agaricus* (*Psalliota*) may show the same peculiarity. This suggestion is justified by the situation in *Agaricus rodmani*. The same situation exists in *Armillaria mellea*.[17]

[16] Atkinson, Geo. F., "The Development of *Agaricus compestris*," *Bot. Gaz.*, **42**: 241–264, pls. 7–12, 1906.

[17] Atkinson, Geo. F., "The Development of *Armillaria mellea*," *Myc. Centralb.*, **4**: 113–121, pls. 1, 2, 1914.

[15] Universal Veil' in *Agaricus*," *Myc. Centralb.*, **2**, 13–19 pls. 1–3, 1914.

In the specimens of *Agaricus arvensis*[18] studied, the lagging behind of the ground tissue below the zone where the hymenophore primordium arises occurs before any differentiation of this zone is distinguishable, for a light area with a looser mesh occurs in an annular zone which marks the distinction between the stem and pileus areas. Or the lagging behind of the ground tissue may occur simultaneously with the appearance of the primordial hymenophore zone and the outline of the pileus area. In a number of forms studied by Fayod[19] the primordium of the pileus is organized, in the apex of the young homogeneous basidiocarp, as a new zone of growth, in the form of an inverted bowl, shown by the darker staining of the hyphæ rich in protoplasm, forming a pileus producing layer ("couche piléogène"). This method of differentiation he accepts as a general law for the Agariceæ, the only exception admitted by him being the coriaceous forms of *Lentinus*. *Agaricus rodmani*, the commercial varieties of *Agaricus campestris* (*columbia* and *alaska*) and *Stropharia ambigua* (Peck) Zeller,[20] also form exceptions to this rule. The primordium of the pileus in these forms may be regarded as diffuse within the upper part of the young basidiocarp, the differentiation and organization of the pileus margin beginning in conjunction with the organization of the primordial hymenophore zone, though in *Stropharia ambigua* the inverted bowl-shaped zone of new growth in the upper part of the pileus area is soon organized.[20] Other forms recently investigated which conform to the general law laid down by Fayod, are certain species of *Hypholoma* (Allen),[21] *Hypholoma fasciculare* and *Clitocybe laccata* by Beer,[22] *Lepiota*[23] *clypeolaria* and *Amanitopsis vaginata*.[24]

[18] Atkinson, Geo. F., "The Development of *Agaricus arvensis* and *A. comtulus*," *Am. Jour. Bot*, 1, 3–22, pls. 1, 2, 1914. "Homology of the 'Universal Veil' in *Agaricus*," *Myc. Centralbl.*, 2, 13–19 pls. 1–3, 1914.

[19] Fayod, V., "Prodrome d'une histoire naturelle des Agaricinées," *Ann. Sci. Nat. Bot.*, VII., 9, 181–411, pls. 6, 7, 1889.

[20] Zeller, S. M., "The Development of *Stropharia ambigua*," *Mycologia*, 6, 139–145: pls. 124, 125, 1914.

[21] Allen, Caroline L., "The Development of some Species of *Hypholoma*," *Ann. Myc.*, 4, 387–394, pls. 5–7, 1906.

[22] Beer, R., "Notes on the Development of the Carpophore in Some Agarincaceæ," *Ann. Bot.*, 25^2: 683–689, pl. 52, 1911.

[23] Atkinson, Geo. F., "The Development of *Lepiota clypeolaria*," *Ann. Myc.*, 12, 346–356, pls. 13–16, 1914.

Organization of the Pileus.—The organization of the pileus begins in connection with the primordial hymenophore zone. The upper part of this zone is very probably to be regarded as the primordium of the pileus margin which then increases by centrifugal growth. It is marked from an early period by strong epinastic growth, so the margin becomes strikingly involute, a feature also characteristic of *Agaricus campestris,*[25] *A. arvensis,*[26] *A. comtulus,* etc., as I have earlier described. The general relation of the hyphæ in the primordium of the pileus margin is a parallel one, and they become more and more strongly incurved as a result of epinasty. As the pileus primordium increases in width by marginal growth, it also increases in thickness, more perceptibly so farther back from the margin where the new growth is older. In this way the organization of the pileus advances more and more into the outer zone of the ground tissue, the blematogen, and becomes consolidated with it.[27]

Organization of the Stem.—The stem area is delimited at the same time as the pileus area by the origin of the young hymenophore zone, but its organization and differentiation from the ground tissue seems to lag behind the early stages of the organization of the pileus margin. While a general and more or less diffuse growth and expansion occurs for some time in the stem area, the first evidence of a differentiation from the ground tissue is seen in the organization of the stem surface. The outline of the stem may be compared to that of a broad, flat cone, since the stem at first is very short and

[24] Atkinson, Geo. F., "The Development of *Amanitopsis vaginata*," *Ann. Myc.*, 12, 369–392, pls. 17–19, 1914.

[25] Atkinson, Geo. F., "The Development of *Agaricus campestris*," *Bot. Gaz.*, 42: 241–264, pls. 7–12, 1906 (see figures 11 and 12).

[26] Atkinson, Geo. F., "The Development of *Agaricus arvensis* and *A. comtulus*." *Am. Jour. Bot.*, 1: 3–22, pls. 1, 2, 1914.

[27] In *Agaricus campestris* var. *edulis,* Vittadini ("Fun. Mang.," 44, pl. 6, fig. 1, 1835) in a young oval fruit body, figures and describes the outline of the pileus within a stout volva, and states that, during the course of development, the volva is ruptured circularly, and the margin of the pileus as it emerges is held for a time against the stem by the lower limb of the annulus. His account of the release of the volva (blematogen) from the pileus does not seem clear, and his figures do not show the transition stage from *a* to *b* in figure 1 of his Plate VI In *Agaricus rodmani* nor in any other species of *Agaricus* (*Psalliota*) have I ever seen any indication of the clear cut outline of the pileus surface as distinct from the blematogen, such as Vittadini shows at *a,* fig. 1.

broad, and the surface slopes outward at a strong angle. The surface outline of the stem is quite clearly differentiated from the loose ground tissue forming the marginal veil, because of the deeper staining property of the stem shown in longitudinal sections (Fig. 32). Its differentiation and organization agrees entirely with that described for *Agaricus campestris,*[28] *Agaricus arvensis* and *A. comtulus.*[29]

Organization of the Marginal Veil.—The organization and limits of the marginal veil, or partial veil, as it is sometimes called, in *Agaricus arvensis, A. comtulus* and *A. campestris,* has been very fully discussed in previous papers[29] (13–15, 1914), briefly in another[30] (17, 1914). Its organization and composition in *Agaricus rodmani* is in the main similar, its different features being due to its more robust character, the stouter pileus and shorter stem. The fundament of the marginal veil is ground tissue in the angle between the primordial hymenophore zone and the stem fundament, including on its outer surface a narrow section of the blematogen layer. The ground tissue in this angle is indicated in *VP* (veil primordium) in diagram 3, and the corresponding areas in Figs. 3, 5, 7, 9–14 can readily be understood. There is considerable increase in this ground tissue by growth of the portion clothing the stem fundament. It is also added to by growth of the hyphæ at the margin of the pileus. The mass of the loose inner surface is often crowded up against the edges of the gills by the involute margin of the pileus pushing it upward, due to epinastic growth.

In such robust specimens usually presented by *Agaricus rodmani* the blematogen layer is comparatively thick but still forms a comparatively small portion of the marginal veil, and lies on the outer under surface of the lower limb of the annulus. By the incurving of the thick margin of the pileus its edge is crowded into the thick veil, and presses against the stem, thus separating the veil, which later becomes the annulus, into an upper and lower limb. As stated above, the fact that the short stem elongates but little in comparison

[28] Atkinson, Geo. F., "The Development of *Agaricus campestris,*" *Bot. Gaz.*, **42**: 241–264, pls. 7–12, 1906.

[29] Atkinson, Geo. F., "The Development of *Agaricus arvensis* and *A. comtulus,*" *Am. Jour. Bot.*, 1. 3–22 pls. 1, 2, 1914.

[30] Atkinson, Geo. F., "Homology of the Universal Veil in *Agaricus, Myc. Cantralb.*, 5, 13–19, pls. 1–13, 1914.

with that of *Agaricus campestris, arvensis,* and a number of other species, the veil is usually not ripped up from the lower part of the stem as it is in the other species. A thin layer on the stem below the annulus is often cracked into distinct areas or patches, the margins of the areas sometimes being partially exfoliated. The partial exfoliation of the under part of the lower limb of the annulus frequently occurs, and then the lower limb itself has a double edge as described above, and as shown in several of the figures of Plate I. In *Agaricus campestris, arvensis, augustus, subrufescens, placomyces,* and others, the freeing of the lower part of the annulus from the stem is very extensive, since as the stem elongates the veil is ripped off for a considerable distance. In *Agaricus rodmani,* as the pileus expands, the lower limb of the veil clings to the stem, splitting off from the outer surface of the pileus margin as the latter is withdrawn. The inner or upper limb of the veil remains attached to the edge of the pileus margin for a longer time, but is eventually separated.

IV. Origin and Development of the Lamellæ.

Origin of the Gill Salients.—The development of the hymenophore is progressive and centrifugal. As described in the previous section, the primordial hymenophore zone originates in conjunction with the primordium of the pileus margin and lies in the angle separating the stem and pileus areas. The organization of the level palisade zone of the hymenophore from the primordial stage, begins in the older region, *i. e.,* next the stem. The margin of the pileus, primordial hymenophore and palisade zone all progress by growth in a centrifugal direction, the younger, later stages succeeding the earlier. The lamellæ succeed the level palisade zone and arise as downward growing salients of the same. These salients begin next the stem (or in some cases on it). They are regularly spaced and progress in a radial, centrifugal direction. The origin of the salients from the level palisade stage is well shown in Figs. 17–21.

In Figs. 18 and 20, different stages in the origin of the salients are shown. Three gill salients are seen in Fig. 20. At the left side of Fig. 20 is the level palisade. Next it to the right is a very low salient. Continuing to read toward the right, the second and third salients are successively stronger. While the hyphal struc-

ture is not very distinctly shown in in this figure, due to the difficulty of illumination which will produce on the photographic plate the same degree of resolution which can be detected by the eye, still the palisade character is evident. A similar situation is seen in Fig. 18, but the progression in the origin and growth of the salients is to be read from right to left. A somewhat later stage is shown in Fig. 19. Here the hyphal structure is well shown. The palisade character of the exposed surface of the hymenophore is very clearly shown. This figure gives us some suggestion of the factors operating in the formation of the gill salients. The elements of the palisade layer increase by interstitial growth, *i. e.*, by new branches which crowd in between the older ones. At the same time the elongate cells composing the palisade layer increase in diameter. In the primordial stage they passed from the terete tapering condition to the cylindrical form. Now they pass from the cylindrical to the clavate form, as well as increasing somewhat in diameter throughout. This produces a great pressure on the level palisade zone, which if continued, must result in throwing the level palisade layer into folds.

Another factor now comes into play which prevents the palisade layer from being thrown into a series of irregular folds. This is the downward growth, by elongation, of the subadjacent tramal hyphæ, along regularly spaced radial areas, beginning next the stem and proceeding in a centrifugal direction toward the margin of the pileus. These radial areas of subadjacent tramal hyphæ, elongating downwards, push the palisade area downward into corresponding radial salients. These salients are the first evidence of folds or ridges which appear in the young hymenophore. They are the gill salients, and by continued growth form the lamellæ themselves.

Fig. 19 presents another very interesting situation. This is the flaring, or fantailing, of the gill salients very soon after their emergence below the level of the general palisade surface. This is very clearly one of the first results of the release from the pressure to which the elongate cells were subject in the level palisade condition. Another still more interesting feature at this stage is the pressure to which the neutral portion of the level palisade is subjected as a result of this fantailing of the gill origins. The flanks of the young

gill salients thus crowd against the intervening neutral palisade cells, more strongly against their free ends. This presses these intervening, neutral, radiating areas of the original level palisade into the form of ridges which thus alternate with the radiating gill salients. These intervening ridges between the young gill salients are very conspicuous in a corresponding stage of gill development in *Coprinus micaceus* as I have shown in another paper. This situation is a comparatively old stage in the development of the lamellæ and is one of the peculiar features presented by a number of the Agaricaceæ, which led Levine[31] to mistake these intervening ridges between comparatively old gill salients for the first ridges to appear in the hymenophore primordium of *Coprinus micaceus*. These ridges he thought were the first evidence of the gills. The gills were described as arising from the splitting of these first ridges and the union of approximate halves of adjacent ridges to form the gills between them. This matter will be referred to below when another peculiar situation is described which also assisted in leading this author astray.

Relation of the Different Phases of Hymenophore Development in the Young Basidiocarp.—Figs. 17–23 represent different phases of the organization and development of the hymenophore in a single basidiocarp, during an intermediate stage of its development. The relation of these different phases is determined by a study of longitudinal serial sections passing from near the stem to the margin of the pileus. With the exception of Fig. 20, Figs. 17–23 are all from the same plant, selected to represent the relation of different phases of the young hymenophore. The sections from which the photographs were taken were parallel with the axis of the stem, and thus were nearly or quite perpendicular to the hymenophore, or under surface of the pileus. The general plane of the hymenophore, or under surface of the pileus, is slightly arched, but for all practical purposes of this study, the plane is perpendicular to the stem axis, so that the sections are perpendicular to the general hymenophore surface, or plane. Fig. 17 is from a section near the stem, corresponding to line 4 in diagram 6 (diagram 6 is intended to illustrate the situation presented by the figures in Plate 5, but serves to illus-

[31] Levine, M., "The Origin and Development of the Lamellæ in *Coprinus micaceus*," *Am. Jour. Bot.*, 1, 343–356, pls. 39, 40, 1914.

trate also the relations now under consideration). An examination of the relation of line 4, in diagram 6, to the gill salients, the palisade and primordial areas, will assist in making the relation of the phases of the hymenophore presented in Fig. 17 very clear.

In the middle of the figure, or section, the gill salients are cut transversely. On either side of the middle they are cut obliquely, the more so the nearer the palisade area the salients are cut. But when the gill is so young, the structure of an oblique section at this angle is practically the same as in a transection. Since the hymenophore is older next the stem, and progressively younger toward the margin of the pileus, the gill salients are older next the stem, and younger next the palisade area, where they are very low and grade off insensibly into the level palisade zone. Toward the left and right from the middle of such a section as is represented by Fig. 17, the salients become less and less prominent until they grade insensibly into the level palisade zone on either side. In like manner the palisade zone grades to the left and right into the primordial zone, and this into the margin of the pileus, showing practically the same relation, so far as the palisade and primordial zones are concerned, as in a radial section.

Fig. 21 is from a section made near the outer ends of the middle salients, about in the region represented by line 7 in diagram 6. Only a few salients are shown, these are very low, and on either side soon grade insensibly into the palisade zone. Fig. 22 is from a section made in the region indicated by line 8 in diagram 6. Here there are no gill salients (nor any evidence of ridges in the hymenophore), a broad area in the middle is the palisade area, and this grades on either side insensibly into the primoridal area. Fig. 23 is from a section made in the region indicated by line 9 in diagram 6. It is entirely within the primordial zone, near the margin of the pileus. Knowing this relation of the different phases of the hymenophore, one can observe the transition of the primordial phase into the level palisade phase, and this into the phase of the salients. In other words, one can study the method of origin of the lamellæ by a study of the different phases of the gill salients in the area of transition from the palisade zone into the zone of the young gills.

Relation of the Hymenophore to the Stem.—One of the taxonomic characters employed for the genus *Agaricus* (*Psalliota*) is the free condition of the gills from the stem. In *Agaricus campestris,* while the gills are usually free, they are close to the stem, and in some cases are even adnexed to the stem. The same is true of *Agaricus rodmani.* Peck[32] says of the lamellæ,—"free, reaching nearly or quite to the stem. It is possible that in some examples the gills may be broadly attached to the stem fundament at the time of their origin, but become free at maturity by changes in the relation and tensions of the parts during expansion of the plant. That the young lamellæ are sometimes broadly attached around the upper end of the stem fundament has been observed in a number of examples during this study of development. In some examples the attachment of the stem is very broad, in others slight, and in still others the lamellæ are free from the time of their origin.

Deceptive Appearance of Sections near the Stem when the Young Lamellæ are Attached.—In studying the origin of the lamellæ in plants where the hymenophore, from its earliest appearance, is entirely free from the stem, little difficulty is experienced in the interpretation of the situation presented, in case there is a fairly well formed annular cavity prior to the origin of the gill salients. Longitudinal sections next the stem then present the simple situation shown in Fig. 17. But in those cases where the hymenophore primordium extends downward on the outer surface of the stem apex, as shown in Figs. 11 and 12, sections passing from the stem through this portion of the hymenophore, after the origin of the gill salients, present a complicated structure, which may be very confusing unless all the features of the situation are taken into consideration. As stated above the stem axis of the sections from which Figs. 11 and 12 were made is parallel with the longitudinal direction of the plate. In very young basidiocarps, as already described, the stem surface slopes outward at a very strong angle as shown in Fig. 32.

Now, when the gill salients begin to form by downward, or outward, extension of the level palisade, in those cases where the hymenophore primordium extends down on the surface of the stem,

[32] Peck, C. H., N. Y. State Mus. Nat. Hist Rept., **36**, 45, 1885.

the salients first appear over this portion of the hymenophore, because it is the older. The older portion of the salients, therefore, extend outward perpendicular to the stem surface. Since their progression is centrifugal, the salients gradually extend over the angle between stem and pileus where their growth is downward. Since the growth in width of the salients is perpendicular to the surface of the level hymenophore at any point, there are formed, in the cases

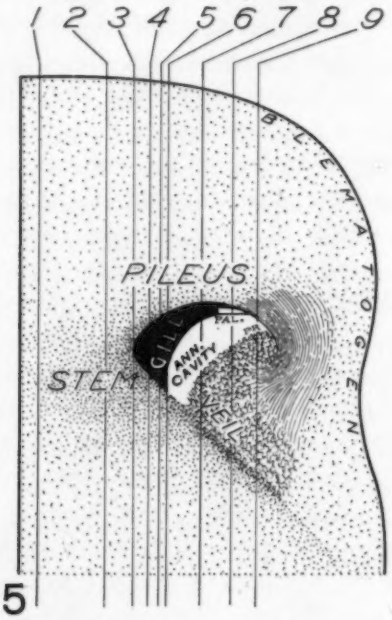


DIAGRAM 5. Lateral view through one half of a basidiocarp in an intermediate stage of development, showing (1) the strongly sloping surface of the stem; (2) the partly organized pileus margin which is becoming involute because of eipnastic growth; (3) the hymenophore presenting three stages of development, (*a*) the oldest portion, the gill area extending on the under side of the pileus and far down on the surface of the stem (adnate at this stage), (*b*) the palisade area (*PAL*) distal to the gill area on under side of pileus, and (*c*) the primordial area (*PR*) near margin of pileus; (4) annular cavity; (5) the loose ground tissue of the marginal veil; and (6) the blematogen layer. See text.

under consideration, a series of little stalls, or pigeon holes, around the stem apex, between the young gills in the angle between the stem

and pileus. This situation is illustrated in Figs. 24–31, from selected serial sections of the same basidiocarp. The sections were parallel with the long axis of the stem. Diagrams 5 and 6 illustrate the situation in this basidiocarp and show exactly how the scetions were made.

Fig. 24 is from a nearly median longitudinal section, made in the region indicated by line 1 of diagrams 5 and 6, which presents a situation practically the same as a median section. The outline of the narrow young gill salient is well shown in Fig. 24, with the distinct annular cavity. The gill salients are strongly curved and in the form of crescents, the lower limb of the crescent extending far down on the outwardly sloping stem surface; the upper limb reaching out on the under surface of the pileus, where it grades into the level palisade zone, and the latter into the primordial zone. The relation of parts is clearly represented by diagram 5. It is quite easy to form a mental picture of the series of little stalls, or pigeon holes, around the upper part of the stem between these crescentic salients.

Fig. 25 is from a section in the region indicated by line 2 of diagrams 5 and 6. The line 2 in diagram 6 shows how the section passes through the side of the stem and obliquely across a few of the young gills, then on either side passing through the level palisade and primordial zones. These features are clearly seen in Fig. 25. Fig. 26 is from the region indicated by line 3: Fig. 27 that of line 4; Fig. 28 that of line 5; Fig. 29 that of line 6; and Fig. 30 that of line 7, of diagrams 5 and 6 (figures of sections in the region indicated by lines 8 and 9 are not shown from this basidiocarp, but there is nothing essentially different in them from figures 22 and 23 from another plant). Fig. 31 is a more highly magnified view of the middle portion of Fig. 27.

Figs. 26–29 and 31 present a very interesting situation. They show transections of the stalls, or pigeon holes, mentioned above. Unless caution is observed this situation would be very misleading. The gill salients are attached above to the under side of the pileus and below to the surface of the stem, and this attachment above and below existed from the time of the origin of the salients. However, the attachment below is not that of the margin of the gills, but of their origin from the stem, since the salients grew outward from the

level palisade organized in this region over the upper surface of the stem.

Similar sections of *Coprinus micaceus*[33] through the region of the attached gills was one of the features contributing to the incorrect interpretation, by Levine, of the origin of the lamellæ in this plant, as shown by his Figs. 13 and 14. The palisade cells on the sides and in the upper angle of these pigeon holes could easily give the impression that the gills had their origin from isolated radial areas of new growth of palisade cells, these areas, or "ridges" of

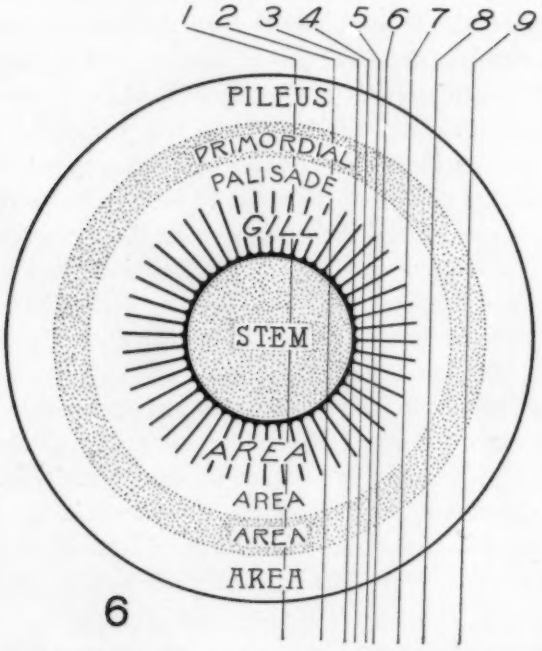


DIAGRAM 6. Zenith view in a basidiocarp of the same age as that represented in diagram 5. See text for details not marked in the diagram.

palisade cells parting as they increase, forming a lining over the ground tissue or partitions of these little stalls, and thus enclosing "the notch between the gills."

Relation of the Gills to the Involute Margin of the Pileus.—There are other peculiar situations presented in the development of

[33] Levine, M., "The Origin and Development of *Coprinus micaceus*," *Am. Jour. Bot.*, 1, 343–356, pls. 124, 125, 1914.

Agaricus rodmani (and other species) which may lead to serious misinterpretation unless great caution is observed. This is the relation of the gills to the involute margin of the pileus and to the marginal veil, shown in a series of longitudinal, "tangential" sections of basidiocarps at an age when the gill salients, by centrifugal progression, have nearly or quite reached the margin of the pileus. The various features of this situation are presented in Figs. 32–42. The figures are photographs of selected serial sections from a single basidiocarp. Diagrams 7 and 8 illustrate the situation in this basidiocarp and the lines show the regions in which the sections were made.

In Fig. 32, from a nearly median longitudinal section (in the region of line 1), the involute margin of the pileus is shown. An indefinite portion of the outer, lighter stained area is the blematogen. The margin of the pileus is so strongly involute that the edge is curved upward toward the gills and has crowded the mass of the ground tissue constituting the inner portion of the veil up against the middle zone of the lamellæ. The attachment of this ground tissue to the margin of the gills is not very firm, though there is some adherence of the hyphæ. The attachment has occurred after the ground tissue was crowded against the margins of the gills by the strongly upturned, involute pileus margin. The strongly involute margin of the pileus is well shown also in several of the figures in Plate VII. The position of the upturned edge of the involute pileus margin is such that the loose ground tissue of the inner portion of the veil is lifted up against the middle area of the lamellæ, while the edges of the gills near the stem and also near the margin of the pileus are free. This is very clearly shown in Fig. 33, from a section in the region of line 2 in diagrams 7 and 8.

Figs. 34 and 35 are from sections in the region of lines 3 and 4 just passing through the surface of the stem in the angle at the junction of the pileus and stem. The hymenophore extends a short distance down on the upper surface of the stem, but the gills are only "adnexed," not extending so far down on the stem fundament as in the basidiocarp represented on Plate XII. and in diagrams 5 and 6. In the middle area of Fig. 35, the nearly solid block of tissue in the same level with the gills on either side, is hymenophore tissue from

the surface of the stem, and a portion of the same area in Fig. 34 also belongs to the hymenophore. The hymenophore, as interpreted here, and in all of my recent papers, includes not only all parts of the lamellæ and the palisade cells between adjacent lamellæ, but also a thin, often indefinite zone of the subadjacent tissue corresponding to the subhymenial tissue of the palisade between the gill origins. As figure 35 shows, the "stalls," or "pigeon holes," in the angle of pileus and stem are quite small because the gill origins extend but

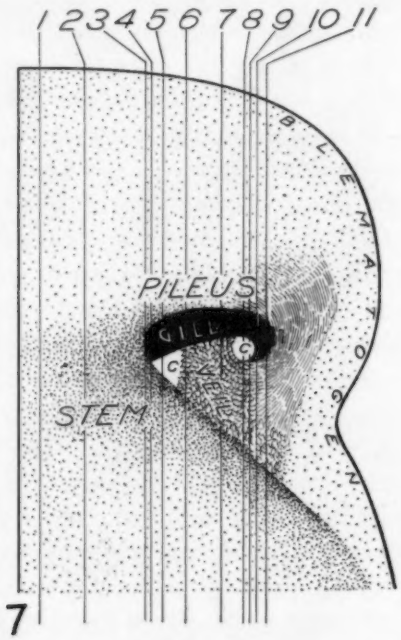


DIAGRAM 7. Lateral view through one half of a basidiocarp in an older stage than that represented in diagrams 5 and 6. The hymenophore has all passed over into the gill stage. The gill area does not extend so far down on the stem as in diagram 5. The margin of the pileus is more strongly involute and the veil tissue has been crowded up against the middle portion of the gills. *C* = the portion of the annular cavity not filled. See text for other details not marked here.

a short distance down on the upper surface of the stem. The abrupt ending of this hymenophore tissue below is even with the margins of the gills on either side, and the lower edge is free from the ground

tissue clothing the stem fundament, as shown by the clear line between the two. This indicates that the portion of the hymenophore on the upper surface of the stem projected by growth slightly above the level of the stem surface, or above that of the ground tissue. In Fig. 34 the distinct boundary line of the more compact tissue shows, but it is in contact with the ground issue below since this section did not pass outside of the junction of stem and pileus fundaments. In Fig. 35 a few of the gills on either side of the middle are free from the ground tissue below. Outside of this on either side (the middle zone between stem and pileus margin) a number of the gills are attached to the ground tissue pressed up against them by the involute pileus margin. On either side of these areas, *i. e.*, near the margin of the pileus, the gills are free.

Fig. 36 is from a section in the region indicated by line 5 in diagram 7. The middle of the section, according to line 5, would pass through the space of the annular cavity near the stem which has not been filled by the upward crowding of the ground tissue. The margin of the gills here should therefore be free from the ground tissue below. This is shown to be the case in Fig. 36, for the gills over the middle portion of the figure (which are near the stem). On either side of this area, however, the section passes through the zone where the ground tissue is crowded up against the gills, while toward the margin of the pileus the gills are again free from the ground tissue.

Figs. 37 and 38 are from sections in the region of lines 6 and 7 respectively, of diagram 7. Both sections are thus "tangents" through the region where the ground tissue in contact with the middle zone of the gills would be continuous and of considerable extent, but the area in the region of line 6 would be of greater extent than that in the region of line 7. This corresponds with the situation shown in Figs. 37 and 38, while toward the margin of the pileus on either side the gills are free. Figs. 39 and 40 are from the region of lines 8 and 9. These pass through the portion of the annular cavity between the margin of the pileus and the ground tissue crowded up against the middle region of the hymenophore. The gills therefore would not be in contact with the ground tissue below. In Figs. 39 and 40, however, it is clear that on either side the gills

are attached below as well as above. The attachment below is not the margin of these gills, but their point of origin from the inner surface of the involute pileus margin. This will be clearly understood from a study of Figs. 41 and 42.

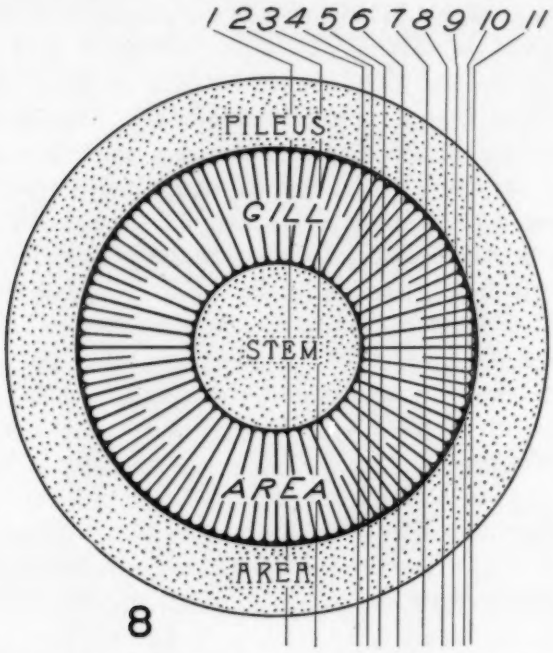


DIAGRAM 8. Zenith view into a basidiocarp of the same age. See text for details not marked.

Figs. 41 and 42 are from sections in the region of lines 10 and 11 in diagrams 7 and 8. The gills are attached above and below. But it is very clear here that the attachment below, as well as above, is to the pileus. Since the gills are downward growths of the level palisade, formed on the under surface of the pileus (*i. e.*, perpendicular to the level palisade), the attachment below in these figures, as well as that above, is at the point of origin of the gills, and must not be interpeted as an attachment of the gill margin to the stem.

The First Ridges, or Salients, of the Hymenophore are the Fundaments of the Lamellæ Themselves.—The question of the origin

of the lamellæ is of renewed interest since it has recently been stated that one of the problems yet to be worked out in the Agaricaceæ is the origin of the lamellæ.[34] The evidence presented in support of this sweeping, and rather surprising statement, is made, so far as we can judge, on the basis of an investigation of *Coprinus micaceus*. It carries with it the implied charge that all of the observations and statements in regard to the origin of the gills, covering a period of more than half a century, are incorrect. In the case of my own work on *Agaricus campestris*,[35] *Armillaria mellea*,[36] *Lepiota clypeolaria*,[37] *Agaricus arvensis*[38] and *A. comtulus* it can be most positively reaffirmed that the lamellæ originate as described, as downward, radial growths of the level palisade portion of the hymenophore. The evidence was so clear in these examples that at the time of the study it did not seem desirable to present full series of "tangential" sections of the different stages in the origin of the gills, particularly as the method of origin agreed in all respects with that described in more than a dozen different species in earlier works. The present study of *Agaricus rodmani* was undertaken, not only for the purpose of examining into the significance of the double annulus, but also for the purpose of examining the different stages in the organization of the hymenophore primordium, the level palisade stage, and the origin of the gills, in a species closely related to *Agaricus campestris*. It is very clear that the present study has fully confirmed the earlier statements with reference to the origin of the lamellæ. Material has also been grown, and the young stages obtained for sectioning in the following commercial forms of *Agaricus: A. campestris* varieties *bohemia* and *alaska*, and *A. "villaticus."*

[34] Levine, M., "The Origin and Development of the Lamellæ in *Coprinus micaceus*," *Am. Jour. Bot.*, 1, 343–356, pls. 39, 40, 1914.

[35] Atkinson, Geo. F., The Development of *Agaricus compestris*," *Bot. Gaz.*, 42, 241–264, pls. 7–12, 1906.

[36] Atkinson, Geo. F., "The Development of *Armillaria mellea*," *Myc. Centralb.*, 4, 113–121, pls. 1, 2, 1914.

[37] Atkinson, Geo. F., "The Development of *Lepiota clypeolaria*," *Ann. Myc.*, 12, 346–356, pls. 13–16, 1914.

[38] Atkinson, Geo. F., "The Development of *Agaricus arvensis* and *A. comtulus*," *Am. Jour. Bot.*, 1, 3–22, pls. 1, 2, 1914. "Homology of the Universal Veil in *Agaricus*," *Myc. Centralb.*, 5, 13–19, pls. 1–3, 1914.

The situation in certain species of *Coprinus*, where the margins of the gills are attached to the stem before maturity, and break away during the expansion of the plants, has for a long time interested me, and I have intended to investigate certain of the species for the purpose of comparing the situation in this genus with that described in *Amanita rubescens*[39] by deBary, *A. muscaria*[40] by Brefeld and in *Amanitopsis vaginata*[41] by myself, where there is no general prelamellar cavity, and the first evidence of the lamellæ is the differentiation of a series of radial trabeculæ in the hymenophore primordium, continuous with the stem and trama of the pileus. This investigation was delayed, however, until the autumn of 1914. Material of three species, *Coprinus comatus, atramentarius* and *micaceus*, was studied, and the results will be published in another paper. This much may he said here, that these three species do not belong to the *Amanita* type but to the *Agaricus* type. There is a strong, annular, prelamellar cavity in *Coprinus comatus*, a weak one in *C. atramentarius* and *micaceus*, but in all three the lamellæ originate as downward-growing salients of a level palisade zone, exactly as described here for *Agaricus rodmani*, the only difference being in those specific features relating to the structure of the lamellæ. Levine based his interpretation of the origin of the lamellæ in *Coprinus micaceus* on complicated and rather well advanced stages of their development. Had the origin of these complicated structures been sought it is probable that the origin of the lamellæ would have been found.

Of the plants thus far studied the following species may be mentioned as examples of the *Agaricus* type in which the origin of the lamellæ has been clearly and correctly described, those by Hoffmann more than half a century ago. *Agaricus carneotomentosus* (*Panus torulosus*) by Hoffmann[42] (1856, p. 145); *Cantharellus*

[39] De Bary, A., "Morphologie und Physiologie der Pilze, Flechten und Myxomyceten," Leipzig, 1866. "Vergleichende Morphologie and Biologie der Pilze, Mycetozoen und Bacterien," 1884. "Comparative Morphology and Biology of the Fungi, Mycetozoa and Bacteria," Oxford, 1887.

[40] Brefeld, O., "Botanische Untersuchungen über Schimmelpilze," 3, Basidiomyceten, I., I.–IV., 1–226; pls. 6–11, 1887.

[41] Atkinson, Geo. F., "The Development of *Amanitopsis vaginata*," *Ann. Myc.*, 12, 369–392, pls. 17–19, 1914.

[42] Hoffmann, H., "Die Pollinarien und Spermatien von Agaricus," *Bot. Zeit.*, 14: 137–148; 153–163, pls. 5, 1856.

tubaeformis, C. aurantiacus, Panus stipticus, Pleurotus tremulus, Omphalia umbellifera, O. pyxidata, Marasmius epiphyllus by Hoffmann[43] (1860); *Collybia velutipes, C. fusipes, Hygrophorus chlorophanus, Galera mycenopsis, Hebeloma mesophaeus, Coprinus fimitarius, Paxillus involutus, Entoloma sericeum,* and others by Hoffmann[44] (1861); *Mycena vulgaris, Collybia dryophila, Nyctalis parasitica, Clitocybe cyathiformis,* and *Cantharellus infundibuliformis* by deBary[45] (1866, 1884, 1887) the latter two in conjunction with Woronin; *Coprinus lagopus* by Brefeld[46] (1877, p. 127); *Agaricus campestris* by Atkinson[47] (1906); *Hypholoma* by Miss Allen[48] (1906) and by Beer[49] (1911); *Stropharia ambigua*[50] by Zeller (1914); *Agaricus arvensis* and *comtulus,*[51] and *Armillaria mellea*[52] by Atkinson (1914).

Summary.

1. The lower limb of the double annulus of *Agaricus rodmani* is not a true volva like that of the Amanitas thus far studied. It is composed of a short segment of the blematogen plus some of the inner tissue of the marginal veil. The greater portion of the blematogen remains "concrete" with or consolidated with the surface of

[43] "Beiträge zur Entwickelungsgeschichte und Anatomie der Agaricinen," *Bot. Zeit.*, **18**: 389–395; 397–404, pls. 13, 14, 1860.

[44] Hoffmann, H., "Icones Analyticæ Fungorum; Abbildungen und Bescreibungen von Pilzen mit besonderer Rucksicht auf Anatomie und Entwickelungsgeschichte, 1–105, pls. 1–24, 1861.

[45] DeBary, A., "Morphologie und Physiologie der Pilze, Flechten und Mycetozoen," Leipzig, 1866. "Vergleichende Morphologie und Biologie der Pilze, Mycetozoen und Bacterien," 1884. "Comparative Morphology and Biology of the Fungi, Mycetezoa and Bacteria," Oxford, 1887.

[46] Brefeld, O., Botanische Untersuchungen über Schimmelpilze," 3, Basidiomyceten, I., I.–IV., 1–226; pls. 6–11, 1887.

[47] Atkinson, Geo. F., "The Development of *Agaricus campestris*," *Bot. Gaz.*, **42**: 241–264, pls. 7–12, 1906.

[48] Allen, Caroline L., "The Development of Some Species of *Hypholoma*," *Ann. Myc.*, **4**: 387–394, pls. 5–7, 1906.

[49] Beer, R., "Notes on the Development of the Carpophore in Some Agaricaceæ," *Ann. Bot.*, **25**[2]: 683–689, pl. 52, 1911.

[50] Zeller, S. M., "The Development of *Stropharia ambigua*," *Mycologia*, **6**: 139–145, pls. 124, 125, 1914.

[51] Atkinson, Geo. F., "The Development of *Agaricus arvensis* and *A. comtulus*," *Am. Jour. Bot.*, **1**: 3–22, pls. 1, 2, 1914.

[52] Atkinson, Geo. F., "The Development of *Armillaria mellea*," *Myc. Centralb.*, **4**: 113–121, pls. 1, 2, 1914.

the pileus, while in *Amanita* the blematogen is finally delimited from the surface of the pileus by a cleavage layer. A double annulus homologous with that of *Agaricus rodmani* is often present in certain other species of *Agaricus*.

2. The primordium of the basidiocarp is oval in form, and homogeneous in structure, consisting of intricately interwoven hyphæ.

3. The four primary parts of the basidiocarp, pileus, stem, marginal veil and hymenophore, are first differentiated by the origin of the hymenophore fundament.

4. The hymenophore primordium arises as an internal, annular zone of new growth toward the upper part of the young basidiocarp. It consists of slender hyphæ rich in protoplasm, parallel, and directed obliquely downward. The lower outer surface is at first more or less open and uneven, presenting a frayed or fimbriate appearance. By continued growth and multiplication of these hyphæ the hymenophore primordium becomes more compact and the under surface becomes even, forming a level palisade zone. Growth of the hymenophore proceeds in a centrifugal direction, the older portions being next the stem fundament. By the epinastic growth of the pileus margin the hymenophore takes on the form of an annular arch.

5. The increase in number and diameter of the elements of the hymenophore fundament produce a tension upon the ground tissue beneath, which lags behind in growth and is torn away from the under surface of the hymenophore, thus forming an annular, prelamellar cavity. This cavity may later be nearly filled by the ground tissue of the inner portion of the veil which increases in bulk, and is often crowded up against the young gills by the involute margin of the pileus.

6. The lamellæ originate as downward growing radial salients of the level palisade zone, beginning next, or on the stem, according as the hymenophore primordium is free from or extends down on the upper portion of the stem fundament. They progress in a centrifugal direction. In an intermediate stage of development or the basidiocarp, all three stages of the hymenophore may be present, the zone of gill salients next the stem, then the level palisade zone, and beyond this the primordial zone.

7. The first ridges, or salients, which appear in connection with the hymenophore are the fundaments of the lamellæ themselves, and the palisade layer is continuous over their edges as well as in the notch between adjacent salients.

DESCRIPTION OF PLATES VII.–XIII.

PLATE VII.

Mature and nearly mature plants of *Agaricus rodmani* showing the double nature of the annulus with its edge grooved; forming an upper and lower limb; the short stem, involute margin of the pileus, etc. × 2/3 diameter. For details see text.

PLATE VIII.

Mature and very robust plants from parking between sidewalk and street. Real size. See text.

PLATES IX.–XIII.

The magnifications of the photomicrographs are as follows: Figs. 3–8; × 9 diameters. Fig. 33; × 10 diameters. Figs. 1, 2; × 12 diameters. Fig. 32; × 13 diameters. Figs. 34–36; × 20 diameters. Fig. 17; × 23 diameters. Figs. 15, 16; × 28 diameters. Figs. 21–30, 37–42; × 30 diameters. Fig. 31; × 100 diameters. Fig. 12; × 110 diameters. Fig. 13; × 155 diameters. Figs. 10, 11; × 160 diameters Fig. 18; × 170 diameters. Figs. 9–14; × 225 diameters. Figs. 19, 20; × 250 diameters.

PLATE IX.

Fig. 1. (No. 18.) Young stage of basidiocarp primordium.

Fig. 2. (No. 20.) Somewhat older stage of basidiocarp primordium, but still in the undifferentiated stage.

Fig. 3. (No. $^{27}/_{1}$.) Earliest stage of differentiation in the young basidiocarp, median longitunial section showing a transection of the internal annular hymenophore fundament, the general prelamellar cavity not yet formed. Pileus fundament is above, stem fundament below, and veil fundament underneath the hymenophore primordium (see Fig. 9).

Fig. 4. (No. $^{27}/_{4}$.) Longitudinal section of the same basidiocarp, "tangential" to the hymenophore primordium, which is shown as a transverse deeply staining area.

Fig. 5. (No. $^{23}/_{1}$.) Median longitudinal section through a basidiocarp just after the formation of the general, annular, prelamellar cavity. The hymenophore is still in the primordial condition (see Fig. 10) but does not extend down on the surface of the upper part of the stem fundament.

Fig. 6. (No. $^{23}/_{3}$.) Longitudinal section of the same basidiocarp, "tangential" to the hymenophore and annular cavity (see Fig. 13).

Fig. 7. (No. $^{12}/_{2}$.) Median longitudinal section of a basidiocarp just after the formation of the general, annular, prelamellar cavity. The

hymenophore is still entirely in the primordial stage (see Fig. 11) and extends for a considerable distance down on the surface of the upper part of the stem fundament.

FIG. 8. (No. $^{12}/_{7}$.) Longitudinal section of the same basidiocarp, "tangential" to the hymenophore and annular cavity (see Fig. 16).

PLATE X.

FIG. 9. (No. $^{27}/_{7}$.) More highly magnified view of the transection of the hymenophore primordium shown in Fig. 3; stem axis at the left. In the darker area (hymenophore primordium) the hyphae extend downward and obliquely outward toward, and some projecting into, the veil fundament below, which consists of a loose mesh of interwoven hyphae.

FIG. 10. (No. $^{23}/_{1}$.) More highly magnified view of the transection of the hymenophore primordium and annular cavity shown in Fig. 5 (axis of stem at the left).

FIG. 11. (No. $^{12}/_{2}$.) More highly magnified view of the transection of the hymenophore primordium and annular cavity shown in Fig. 7 (stem axis at right). The hymenophore primordium extends down over the upper part of the stem outer surface. Veil fundament in the angle below, the ground tissue tearing apart and separting from the fimbriate under surface of the hymenophore

FIG. 12. (No. $^{3}/_{3}$.) Transection of hymenophore and annular cavity, showing same view as Fig. 11 (stem axis at right) but in another basidiocarp and slightly older stage; the portion of the hymenophore primordium on the upper part of the stem fundament has become transformed into the level palisade stage.

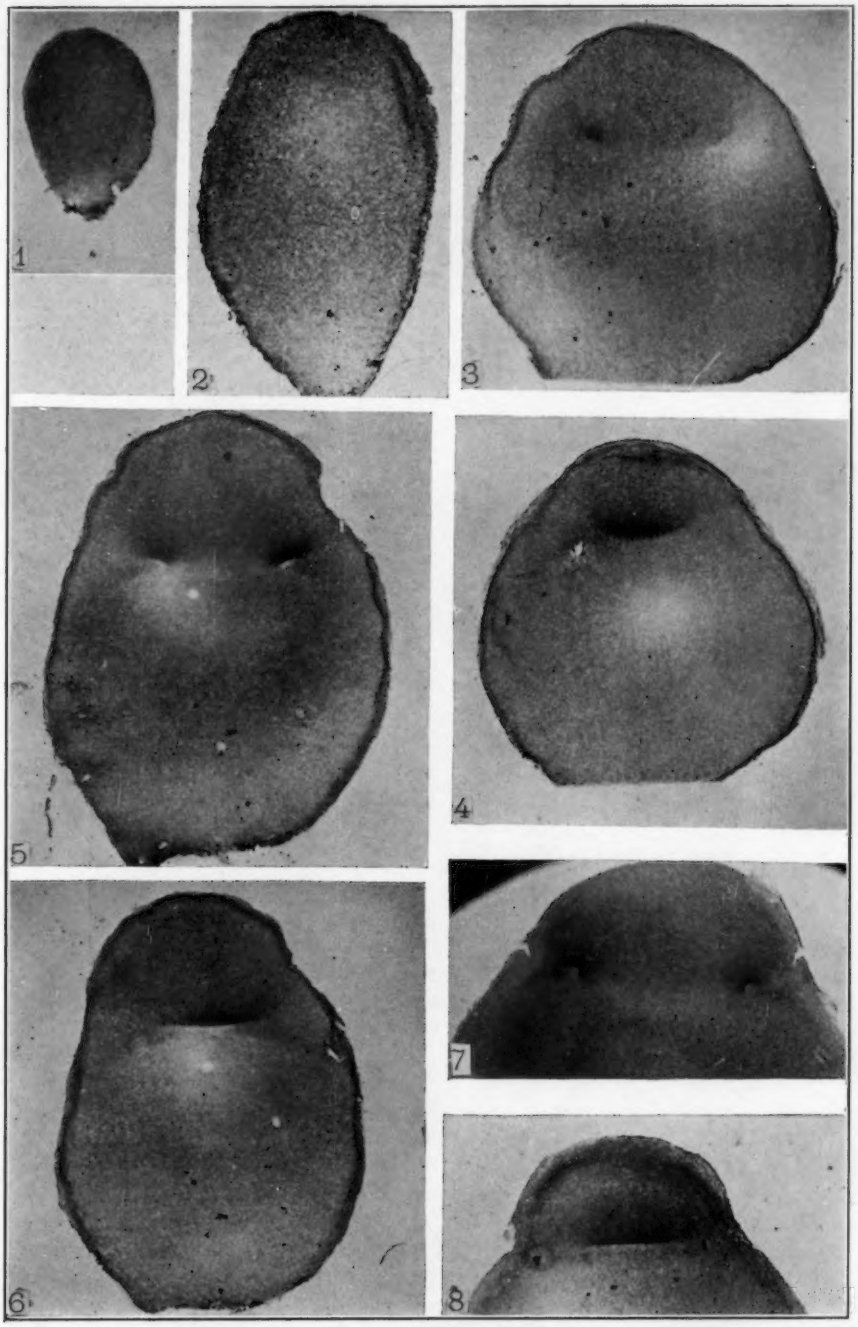
FIGS. 13 and 14. (No. $^{2}/_{2}$.) Section of another basidiocarp showing the hymenophore and annular cavity in same stage as in Fig. 10, at different magnifications (stem axis at right). Hymenophore primordium with fimbriate edge. Ground tissue below (veil fundament) breaking away from the fimbriate surface of the hymenophore as a result of the tension produced by the rapid increase in number and size of the elements of the hymenophore and the lagging behind of the ground tissue below, thus forming the annular cavity. These sections are radial and parallel with the direction of the later lamellæ. The elements of the hymenophore here are somewhat clustered, the slender ends of the hyphæ clinging in groups as the lower surface of the hymenophore is loosened by the tension of the increase above.

FIG. 15. (No. $^{23}/_{3}$.) "Tangential" section of the hymenophore primordium, more highly magnified view of the hymenophore and general, annular, prelamellar cavity shown in Fig. 6. Note the fimbriate lower surface of the hymenophore primordium, and the loose ground tissue (primordium of veil) below separating from it and forming the annular cavity. The structure of the hymenophore primordium is homogeneous, there is not the slightest evidence of gill salients, or of ridges of any sort, which precede or have any relation to the lamellæ which are to arise later.

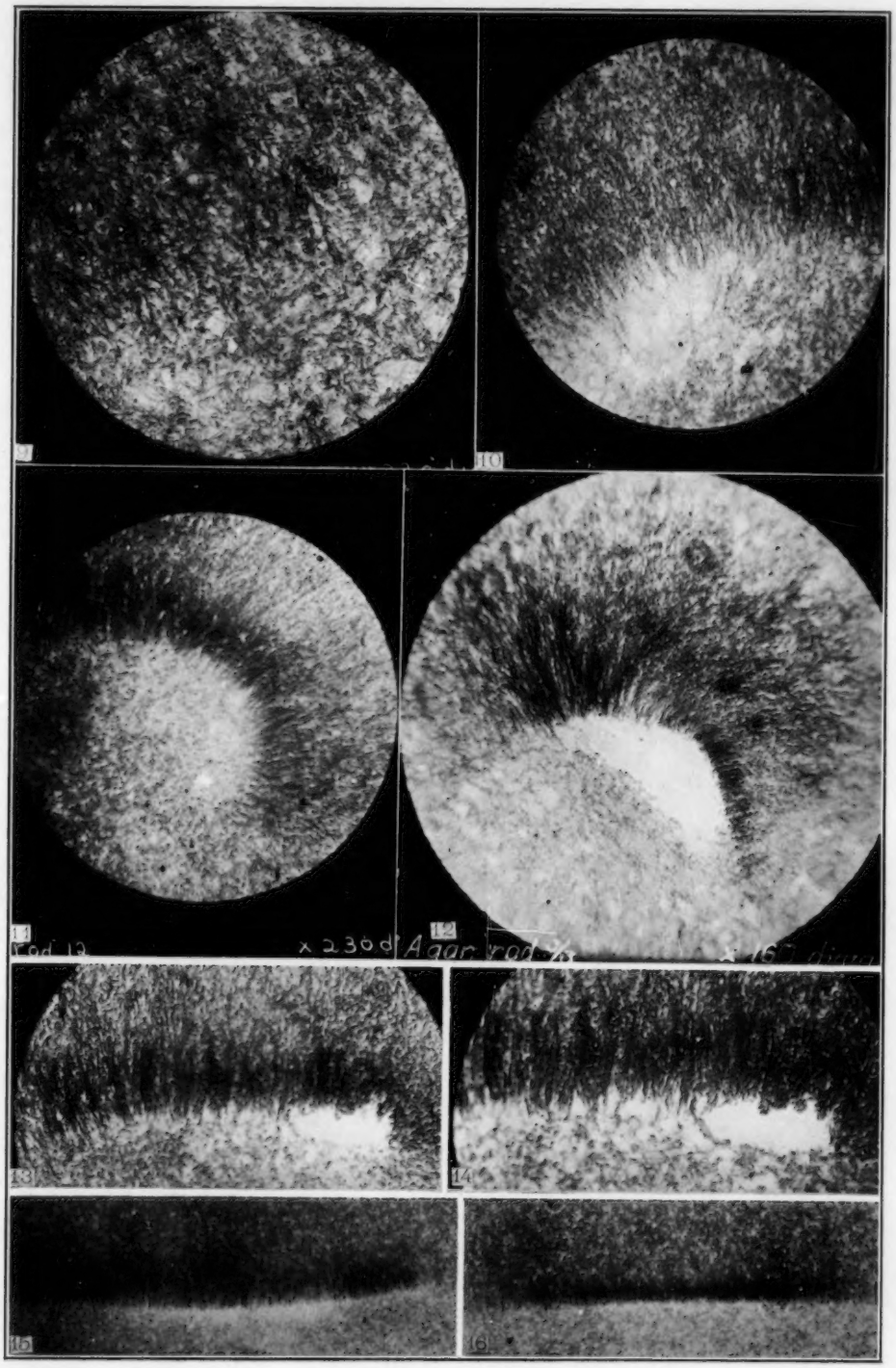
FIG. 16. (No. $^{12}/_{7}$.) "Tangential" section of hymenophore primordium, annular cavity and veil fundament, a more highly magnified view of this part of the basidiocarp shown in Fig. 8. Details as in Fig. 15.

AGARICUS RODMANI

AGARICUS RODMANI



AGARICUS RODMANI



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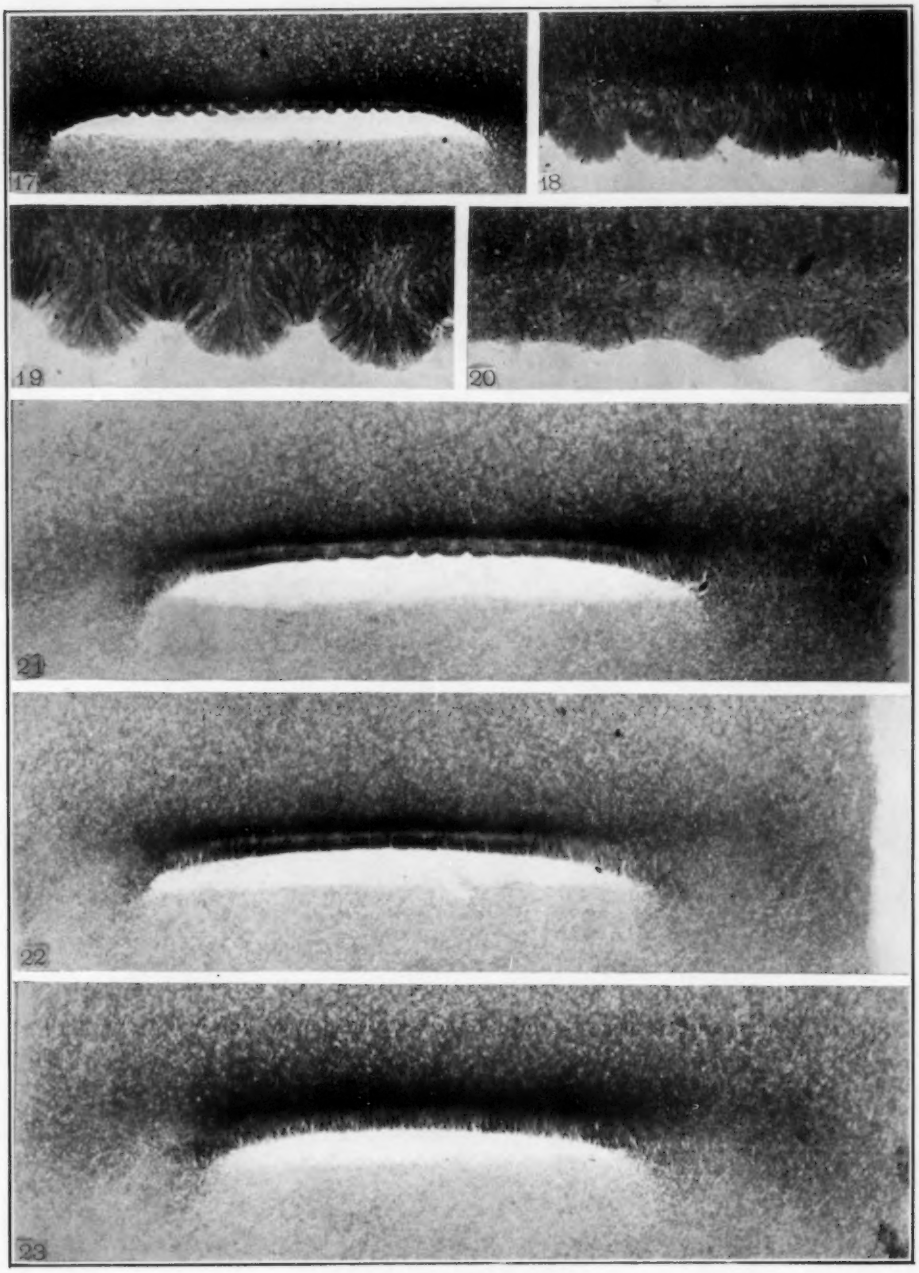

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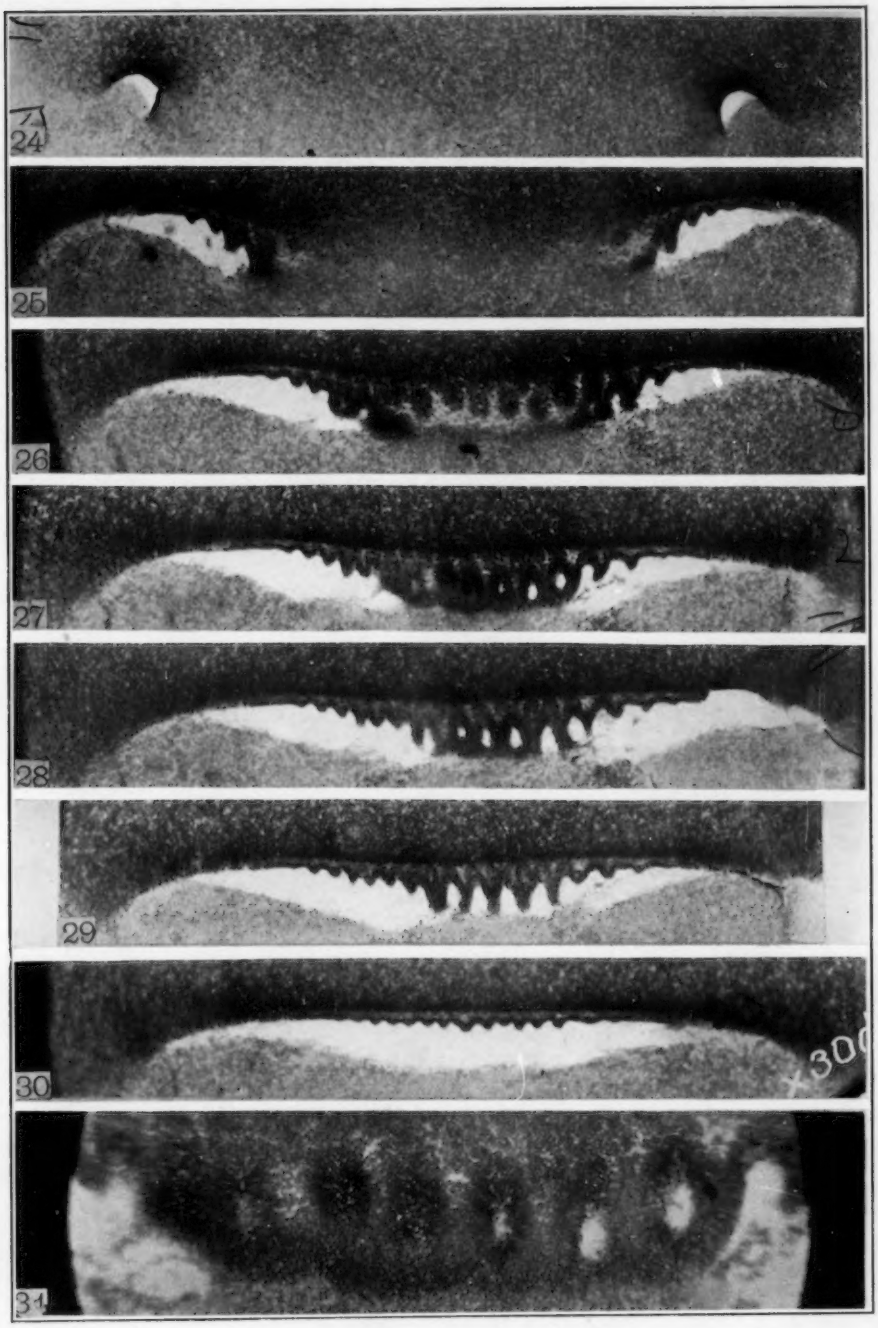

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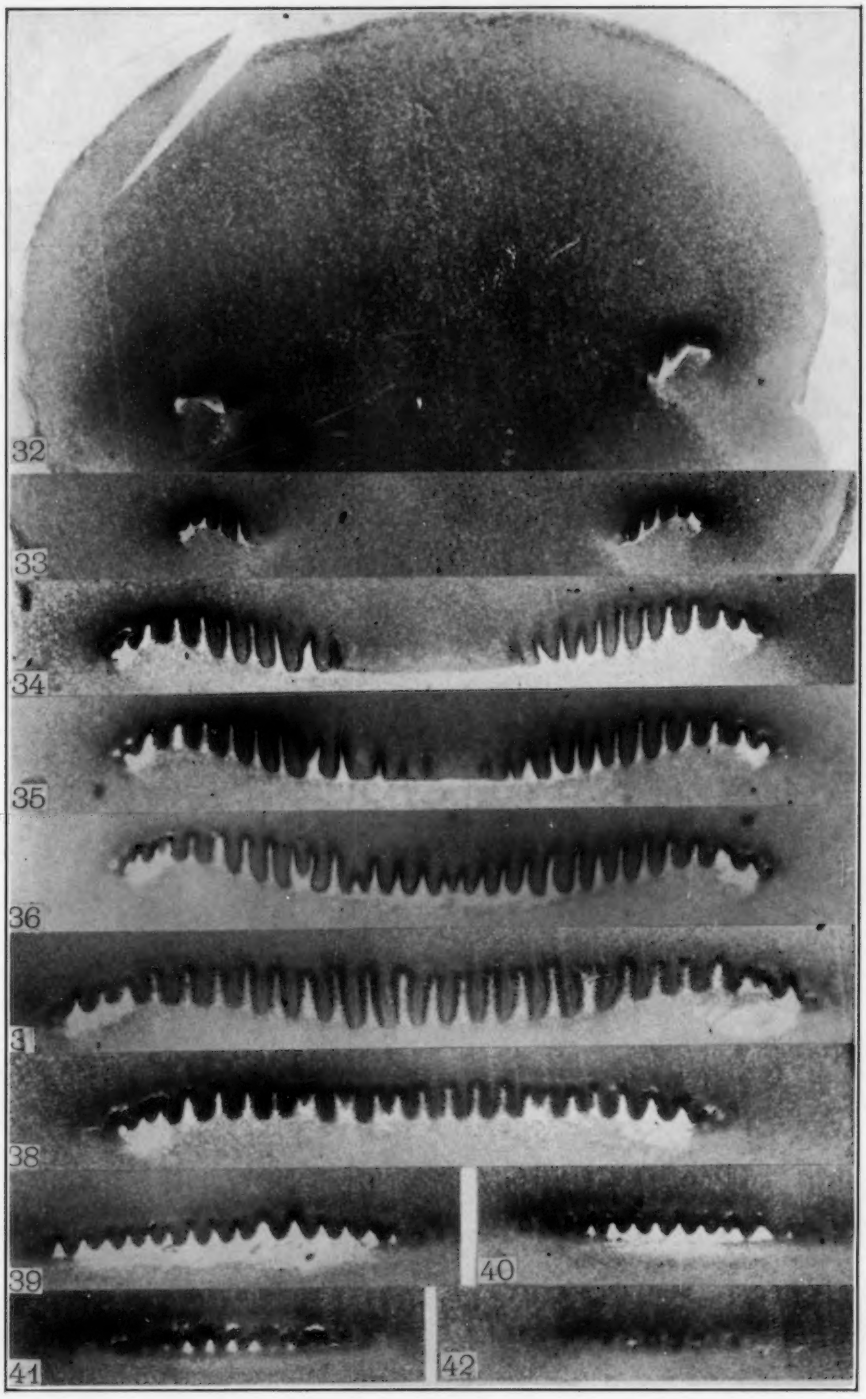

AGARICUS RODMANI

PLATE XI.

FIGS. 17–19 and 21–23, all from a single basidiocarp (No. 5½), from selected serial sections parallel with the axis of the stem and "tangential" in the pileus. Fig. 17 is from near the stem, and shows the three stages of the developing hymenophore, primordial zone, level palisade zone, and the zone of gill salients (transected) with different stages in the origin of the latter from the level palisade condition (see text for details). The general annular cavity is well shown.

FIG. 18. More highly magnified view of portion of the same section in the region of the origin of the gill salients from the level palisade stage.

FIG. 19. More highly magnified view of the young gill salients, showing how they flare, or fantail, when released from the pressure to which the elements are subjected in the level palisade zone, also showing how this flaring of the young gill salients crowds the intervening palisade cells of the original level into "ridges," these ridges of palisade in the notch between two lamellæ being formed later than the gill salients, and as a result of the lateral pressure of the flaring salients. For details see the text.

FIG. 20. (No. ½.) Section from another basidiocarp showing transition from the level palisade stage to the gill salients.

FIG. 21. Section nearer the margin of the pileus than that shown in Fig. 17. In the middle area the gill salients are cut near their distal end where they are very low (see text for details). Transition to level palisade and primordial zone on either side.

FIG. 22. Section still nearer the margin of the pileus showing the level palisade zone in the center, and the primordial zone on either side.

FIG. 23. Section still nearer the margin of the pileus, entirely through the primordial zone.

PLATE XII.

FIGS. 24–31. Selected serial sections from a single basidiocarp (No. ⅓), parallel with the stem axis and from nearly median in the stem to midway from stem surface to the margin of the pileus. Here the hymenophore extends for some distance down on the outward sloping surface of the stem fundament, and there are little "stalls" or pigeon holes between them in the angle at junction of pileus and stem. See text for details.

PLATE XIII.

FIGS. 32–42. Selected serial sections from a single basidiocarp (No. 11), parallel with the axis of the stem and from median in the stem to "tangential" in the margin of the pileus. See text for details.

PROC. AMER. PHIL. SOC., LIV. 219 W, PRINTED SEPT. 8, 1915.

THE EULER–LAPLACE THEOREM ON THE DECREASE OF THE ECCENTRICITY OF THE ORBITS OF THE HEAVENLY BODIES UNDER THE SECULAR ACTION OF A RESISTING MEDIUM.

By T. J. J. SEE.

(*Read April 24, 1915.*)

In the "Mécanique Céleste," Liv. VII., Chap. VI., §§ 29–30, and Liv. X., Chap. VII., § 18, Laplace has developed the mathematical theory of the secular action of a resisting medium, and applied it to the motions of the moon and planets. The first discussion herein cited was published in Volume III. of the "Mécanique Céleste," 1802. It is on this discussion by Laplace that modern investigators chiefly base their treatment of the problems of a resisting medium. Laplace's development of the theory therefore has been of great service to science for more than a century.

Recently, while occupied with a careful review of the theories of magnetism and of gravitation since the time of Newton, I had occasion to examine Euler's "Dissertatio de Magnete," 1744, "Opuscula," 1746–51; and while looking into this work was surprised to find that Euler had preceded Laplace in his development of the chief effects of a resisting medium by more than half a century. Euler's work on the resisting medium will be found in the volume of "Opuscula," Berlin, 1746, in the paper "De Relaxatione Motus Planetarum," pp. 245–276.

Having shown that the aphelia are undisturbed by resistance, Euler considers in section XVII. the equations for the mean motion, and the return to perihelion, after changes in the mean motion by the increments representing a whole revolution:

$$nt + 2\pi, \quad nt + 4\pi, \quad nt + 6\pi, \quad nt + 8\pi, \text{ etc.}$$

Euler puts for the planetary orbit about the sun,

$h = a(1-e)$ = perihelion distance,

$g = a(1-e^2) = p$ = latus rectum of the orbit,

$y = r$ = radius vector of the planet,

$Z = e$ = the eccentricity of the orbit,

t = true anomaly = v, in the notation now commonly used,

s = arc of the orbit, reckoned from perihelion,

c = sun's mean distance,

= $\mu\alpha$, where α is the earth's equatorial semi-diameter, and

μ a number which expresses the sun's mean distance in this unit. Euler uses a solar parallax of 13″, and takes $c = 15866\alpha$. With the values now adopted in astronomy we have about $c = 23445\alpha$. In some of his numerical work Euler uses $c = g = a(1-e^2)$, which is admissible when we neglect the square of the eccentricity.

Euler also uses a small angle of deviation due to the angular effects of resistance, $z = \theta$, such that $\tan z = 2g/3c$; and then takes the equation for the Keplerian ellipse

$$r = \frac{a(1-e^2)}{1+e\cos v},$$

to have the form of an ellipse modified by resistance

$$\frac{1}{r} = \frac{1}{p}(1+e\cos v) = \frac{1}{p} + \frac{e\cos v}{p} + P,$$

where P is function of the time, but modified by a very small quantity depending on the effects of the secular action of the resisting medium.

From the equations of the disturbed ellipse, in his notation,

$$\frac{1}{y} = \frac{1}{g} + \frac{\zeta}{g}\cos t + P,$$

$$P = \frac{1}{c}(t - \sin t - \tfrac{1}{2}\zeta \sin t + \tfrac{1}{2}\zeta t\cos t + \tfrac{3}{4}\zeta\zeta t - \tfrac{2}{3}\zeta\zeta \sin t - \tfrac{1}{24}\zeta\zeta \sin 2t),$$

Euler develops the following table:

If	there will be
$t = 0,$	$P = 0,$
$t = \pi - \theta,$	$P = \frac{1}{c}\left(\pi - \frac{1}{2}\zeta\pi - \frac{4g}{\zeta c}\right),$
$t = 2\pi,$	$P = \frac{1}{c}(2\pi + \zeta\pi),$
$t = 3\pi - \theta,$	$P = \frac{1}{c}\left(3\pi - \frac{3}{2}\zeta\pi - \frac{4g}{\zeta c}\right),$
$t = 4\pi,$	$P = \frac{1}{c}(4\pi + 2\zeta\pi),$
$t = 5\pi - \theta,$	$P = \frac{1}{c}\left(5\pi - \frac{5}{2}\zeta\pi - \frac{4g}{\zeta c}\right).$

He remarks that when therefore for perihelion we have

$$\frac{1}{y} = \frac{1+\zeta}{g} + P,$$

and for aphelion

$$\frac{1}{y} = \frac{1-\zeta}{g} + P,$$

if	
$t = 0,$	$\frac{1}{y} = \frac{1+\zeta}{g} + 0,$
$t = \pi - \theta,$	$\frac{1}{y} = \frac{1-\zeta}{g} + \frac{\pi}{c}(1 - \frac{1}{2}\zeta),$
$t = 2\pi,$	$\frac{1}{y} = \frac{1+\zeta}{g} + \frac{2\pi}{c}(1 + \frac{1}{2}\zeta),$
$t = 3\pi - \theta,$	$\frac{1}{y} = \frac{1-\zeta}{g} + \frac{3\pi}{c}(1 - \frac{1}{2}\zeta),$
$t = 4\pi,$	$\frac{1}{y} = \frac{1+\zeta}{g} + \frac{4\pi}{c}(1 + \frac{1}{2}\zeta),$
$t = 5\pi - \theta,$	$\frac{1}{y} = \frac{1-\zeta}{g} + \frac{5\pi}{c}(1 - \frac{1}{2}\zeta);$

it being understood that the final angle $4g/3c$ is neglected as very small.

Euler next considers the effect of i whole revolutions:

$$t = 2i\pi,$$

$$\frac{1}{y} = \frac{1+\zeta}{g} + \frac{2i\pi}{c}(1+\tfrac{1}{2}\zeta);$$

and finds for the radius vector:

$$y = \frac{g}{1+\zeta} - \frac{2i\pi}{c}\frac{(1+\tfrac{1}{2}\zeta)gg}{(1+\zeta)^2}.$$

Putting for the following aphelion, $t = (2i+1)\pi - \theta$, there will result

$$\frac{1}{y} = \frac{1-\zeta}{g} + \frac{(2i+1)\pi}{c}(1-\tfrac{1}{2}\zeta);$$

whence the radius vector becomes

$$y = \frac{g}{1-\zeta} - \frac{(2i+1)\pi(1-\tfrac{1}{2}\zeta)gg}{c(1-\zeta)^2}.$$

The successive distances of the planet from the sun are diminished in the following manner:

I. Perihelion	$\frac{g}{1+\zeta} = 0,$
Aphelion	$\frac{g}{1-\zeta} - \frac{\pi(1-\frac{1}{2}\zeta)gg}{c(1-\zeta)^2},$
II. Perihelion	$\frac{g}{1+\zeta} - \frac{2\pi(1+\frac{1}{2}\zeta)gg}{c(1+\zeta)^2},$
Aphelion	$\frac{g}{1-\zeta} - \frac{3\pi(1-\frac{1}{2}\zeta)gg}{c(1-\zeta)^2},$
III. Perihelion	$\frac{g}{1+\zeta} - \frac{4\pi(1+\frac{1}{2}\zeta)gg}{c(1+\zeta)^2},$
Aphelion	$\frac{g}{1-\zeta} - \frac{5\pi(1-\frac{1}{2}\zeta)gg}{c(1-\zeta)^2},$ etc.

In any revolution about the sun the perihelion advances by the interval

$$\frac{2\pi(1+\frac{1}{2}\zeta)gg}{c(1+\zeta)^2};$$

and the aphelion regresses by the interval

$$\frac{2\pi(1-\frac{1}{2}\zeta)gg}{c(1-\zeta)^2};$$

the mean distance therefore decreases in the interval about $2\pi gg/c$; and after i revolutions this decrease in the mean distance will be $2i\pi gg/c$.

Accordingly, after i planetary revolutions, the perihelion distance from the sun becomes:

$$\frac{g}{1+\zeta}-\frac{2i\pi(1+\frac{1}{2}\zeta)gg}{c(1+\zeta)^2};$$

and the following aphelion distance:

$$\frac{g}{1-\zeta}-\frac{(2i+1)\pi(1-\frac{1}{2}\zeta)gg}{c(1-\zeta)^2}.$$

The addition of these values, after i revolutions, effects the transverse axis of the orbit:

$$\frac{2g}{1-\zeta\zeta}-\frac{4i\pi gg}{c(1-\zeta\zeta)^2}-\frac{\pi(1-\frac{1}{2}\zeta)gg}{c(1-\zeta)^2}.$$

Here indeed, since the time is to be defined, the time from perihelion to aphelion may be omitted; and thus after i revolutions the transverse axis of the orbit is found to be:

$$\frac{2g}{1-\zeta\zeta}-\frac{4i\pi gg}{c(1-\zeta\zeta)^2};$$

wherefore also the initial transverse axis is assumed equal to $2g/(1-\zeta\zeta)$.

If, therefore, the distance from the perihelion to the sun after i revolutions, which is equal to

$$\frac{g}{1+\zeta}-\frac{2i\pi(1+\frac{1}{2}\zeta)gg}{c(1+\zeta)^2},$$

be subtracted from the distance from the aphelion to the sun, which would develop in the same time, and found to be equal to

$$\frac{g}{1-\zeta} - \frac{2i\pi(1-\frac{1}{2}\zeta)gg}{c(1-\zeta)^2},$$

it will give for the distance of the foci after i revolutions

$$\frac{2\zeta g}{1-\zeta\zeta} - \frac{2i\pi\zeta gg(3-\zeta\zeta)}{c(1-\zeta\zeta)^2}.$$

The initial transverse axis was $2g/(1-\zeta\zeta)$, and if we divide this into the last expression, we get for the eccentricity of the orbit at this time: $\zeta - 3i\pi\zeta g/c$, terms in ζ^3 being neglected as insensible.

In Euler's paper the factor 3 in the last term is inadvertently omitted. He remarks that the original eccentricity was ζ, whereas after i revolutions it is decreased by the negative term shown above, and thus is subject to a secular diminution, owing to the secular action of the resisting medium.

After this discussion Euler reaches the conclusion: "A resistentia ergo excentricitas continuo minuitur, orbitaeque planetarum propius ad figuram circularem reducuntur" (p. 271).

He therefore recognized clearly that the effect of a resisting medium is to decrease the eccentricity incessantly, and to render the orbit more and more circular; and had reached this important conclusion some fifty-six years (1746) before the corresponding theorem was established by Laplace in 1802.

Accordingly as Euler's reasoning is essentially rigorous, though not the same as that of Laplace, it is evident that he was the first discoverer of the theorem which is of such fundamental importance in the theories of cosmogony.

It is remarkable that although Laplace had this theorem clearly before his mind for a quarter of a century at the close of his life (1802–1827) he did not once suspect that the planets and satellites had originated in the distance and through the action of a resisting medium had neared the centers about which they now revolve, and thus acquired the wonderful circularity of their orbits.

It is well known that Laplace continually refers to these bodies as detached by rotation, in the form of zones of vapor, as first

outlined in his nebular hypothesis of 1796. He thus misled the scientific world for more than a century, till the capture theory, involving formation in the distance with subsequent approach to their central masses, under the secular action of a resisting medium, was developed by the present writer in 1908–10.

It is equally well known that Laplace always held the comets to be foreign to our system—another misleading doctrine in cosmogony, finally overthrown in 1910 by the independent researches of Strömgren of Copenhagen, and the present writer, who showed that the comets are surviving residues of the ancient nebula which formed our solar system.

In my "Researches," Vol. II., pp. 138–139, I have drawn attention to two letters from Euler to the Royal Society, pointing out, as early as 1749, that the earth was once beyond the present orbit of Saturn. He does not there discuss the secular decrease of the eccentricity of the planetary orbits; yet as he had grounds for holding to a secular approach to the central masses, he was the first writer to outline sound views in cosmogony.

Under the circumstances it appears appropriate that the theorem on the secular decrease of the eccentricities of the orbits of bodies moving in resisting media, should be known by the name of the Euler-Laplace theorem. This recognizes the correct historical development, as now made out; and probably will always hold a fundamental place in the science of celestial evolution.

Mare Island, California,
April 6, 1915.